A Review and Taxonomy of Distortion-Oriented Presentation Techniques

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One of the common problems associated with large computer-based information systems is the relatively small window through which an information space can be viewed. Increasing interest in recent years has been focused on the development of distortion-oriented presentation techniques to address this problem. However, the growing number of new terminologies and techniques developed have caused considerable confusion to the graphical user interface designer, consequently making the comparison of these presentation techniques and generalization of empirical results of experiments with them very difficult, if not impossible. This article provides a taxonomy of distortion-oriented techniques which demonstrates clearly their underlying relationships. A unified theory is presented to reveal their roots and origins. Issues relating to the implementation and performance of these techniques are also discussed.

Categories and Subject Descriptors: H.5.2 [**Information Interfaces and Presentations**]: User Interfaces

General Terms: Human Factors

Additional Key Words and Phrases: Bifocal displays, distortion-oriented presentation, fisheye views, focus + context techniques, graphical interfaces, information visualization, Perspective Wall, presentation techniques

1. INTRODUCTION

One of the common problems associated with large computer-based information systems is the relatively small window through which an information space can be viewed. This gives rise to problems (i) in locating a given item of information (navigation), (ii) in interpreting an item, and (iii) in relating it to other items, if the item cannot be seen in its full context. Various techniques have evolved for accessing large volumes of data through a limited display

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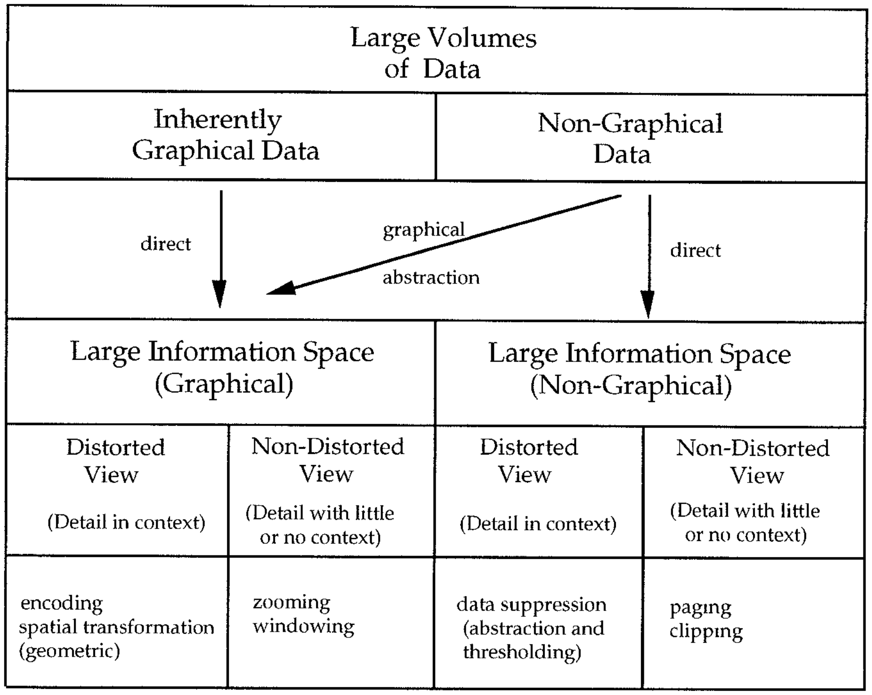

Fig. 1. A taxonomy of presentation techniques for large graphical data spaces.

surface, and these can be broadly categorized as distortion-oriented and nondistortion-oriented presentations. The data itself can also be classified according to whether it is inherently graphical in nature, with implicit spatial relationships, or whether it is nongraphical—although in many cases data of this latter type can be represented in an abstract graphical form [Leung and Apperley 1993b]. Figure 1 shows a simple taxonomy of these techniques, with examples of each of the four types.

Nondistortion-oriented techniques have been used quite some time for the presentation of textual data [Monk et al. 1988; Beard and Walker 1990] and in a number of graphical applications [Donelson 1978; Herot et al. 1980; Leung 1989]. The most familiar approach is simply to display a portion of the information at a time, and to allow scrolling or paging to provide access to the remainder. An alternative, and one which does enhance the ability to find a specific item of information, is to divide the total information space into portions which can be displayed, and to provide hierarchical access to these "pages"; as one moves down the hierarchy then more detailed information is given about a smaller area of the information space. Another approach, which exploits specific structure in the data (in this case a tree structure), involves arranging or representing the data in a special way for presentation, as a Tree-Map [Johnson and Shneiderman 1991; Shneiderman 1992] or as a Cone Tree [Robertson et al. 1991].

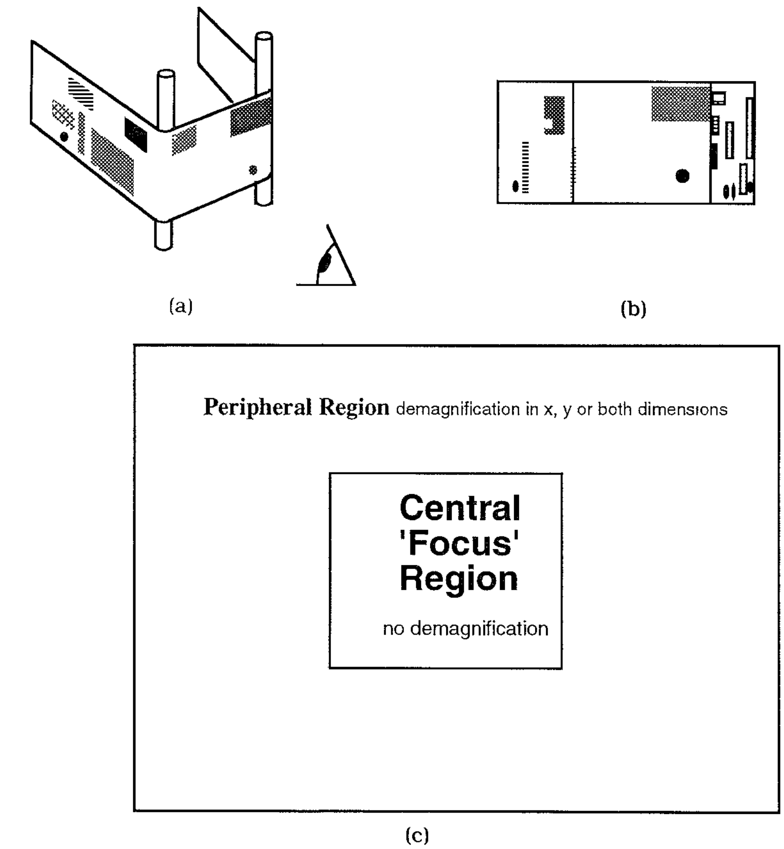

Fig. 2. (a) A mechanical model of a distortion-oriented presentation technique. This model characterizes both the Perspective Wall and the Bifocal Display; (b) the appearance of the data space transformed by a distortion-oriented presentation technique, in this case the Bifocal Display, obtained by viewing the model in (a) from infinity; (c) the presentation of a 2D distortion technique.

While nondistortion-oriented techniques may be adequate for small text-based applications, their main weakness is that generally they do not provide adequate context for the user to support navigation of large-scale information spaces. To overcome this shortcoming, distortion-oriented techniques have been developed and used, particularly in graphical applications. The main feature of these techniques is to allow the user to examine a local area in detail on a section of the screen, and at the same time, to present a global view of the space to provide an overall context to facilitate navigation (see Figure 2).

The growing interest in the application of distortion techniques in recent years [Leung 1989; Hollands et al. 1989; Mackinlay et al. 1991; Misue and Sugiyama 1991; Sarkar and Brown 1992; Robertson and Mackinlay 1993; Rao and Card 1994] can be attributed to the availability of low-cost and high-performance graphics workstations. Farrand [1973] provided an early discussion of computer-based application of distortion-oriented display techniques. He considered the graphical fisheye and designed his DECR (Detail Enhancing, Continuity Retaining) lens to address what he termed the DETAIL × SCOPE problem in information display. In the context of the noninteractive presentation of cartographic maps, Kadmon and Shlomi [1978] described the Polyfocal Display. Kadmon and Shlomi laid down the mathematical foundation for a variety of distortion techniques, and they also proposed the concept of a multifocal projection.

The Bifocal Display [Spence and Apperley 1982] was an early computer-based distortion-oriented display technique. The original illustration of the Bifocal Display was a one-dimensional representation of a data space whose area exceeded that of the screen; the example used was an "in-tray" coupled with an application for an office environment. The Bifocal Display was extended later to a two-dimensional form for the presentation of topological networks [Leung 1989]. A variant of the Bifocal Display in one-dimensional form was proposed later by Mackinlay et al. [1991] as the Perspective Wall.

Furnas' concept of a Fisheye View [Furnas 1986] was based on textual trees, and an implementation of this technique was illustrated in the presentation of program code in one-dimensional form and a calendar in two-dimensional form. No mathematics for the graphical application of this concept were provided. A number of Fisheye View-like applications have been developed [Hollands et al. 1989; Mitta 1990; Misue and Sugiyama 1991; Sarkar and Brown 1992; Schaffer et al. 1993] which differ not only in their application domains, but also in their form. While Sarkar and Brown [1992] attempted to formalize the mathematical foundation for the Fisheye View, their illustration of the technique applied to topological networks was based on a variation of the ideal Fisheye View.

The fast growing number of distortion-oriented techniques proposed by user interface designers calls for a taxonomy and a unified theory to relate and delineate these techniques for two main reasons. First, a taxonomy will help to clarify the confusion of terminologies and unravel the mystique of ever-increasing new presentation techniques confronting graphical user interface designers. Second, a well-defined classification will help to make the comparison and generalization of empirical results of experiments using these techniques a much easier task.

The main aims of this article are fourfold: (i) it reviews distortion-oriented presentation techniques reported in current literature and explains their fundamental concepts, (ii) it presents a taxonomy of these techniques clearly showing their underlying relationships, (iii) a unified theory of distortion-oriented techniques is presented to show their roots and origins, and (iv) issues relating to the implementation and performance of these techniques are discussed.

2. A REVIEW OF DISTORTION-ORIENTED PRESENTATION TECHNIQUES

The application of distortion-oriented techniques to computer-based graphical data presentation has a relatively short history, although the concept of distortion or deformation has been used over many centuries by cartographers in various map projections. Modern distorted displays can be found in familiar representations as the London Underground map and many subsequent subway systems and topological networks.

The essence of these techniques is the concurrent presentation of local detail together with global context at reduced magnification, in a format which allows dynamic interactive positioning of the local detail without severely compromising spatial relationships. Figures 2(a) and 2(b), show a mechanical analogy of a simple distortion technique (the Bifocal Display) applied in one dimension on a strip of graphical information. An illustration of a general two-dimensional distortion-oriented technique is shown in Figure 2(c). With these types of techniques there is usually a focus region where detailed information is displayed; in its surrounding regions, a demagnified view of the peripheral areas is presented.

A distorted view is created by applying a mathematical function, which is called a transformation function, to an undistorted image. The transformation function for a presentation technique defines how the original image is mapped to a distorted view. A magnification function, which is the derivative of a transformation function, on the other hand provides a profile of the magnification (or demagnification) factors associated with the entire area of the undistorted image under consideration. Figures 3(a) and 3(b) show the relationship of these two functions and illustrate how an elliptical object is transformed to its distorted form by applying the transformation function of a Bifocal Display in one dimension.

In a real-time system, the user may initiate a shift of the focus region to view an adjacent area in detail using an interaction device. Then the system will apply the transformation function to every entity contained in the repositioned image and update the display with a corresponding shift in the focus region and its contents; the peripheral regions are also updated at the same time. The system response time depends on three factors; the complexity of the mathematical transformations involved, the amount of information and detail to be presented, and the computational power and suitability of the system used for implementation.

The following subsections present a historical review of distortion-oriented techniques and their underlying concepts in chronological order. The general form of their respective transformation and magnification functions is illustrated and applied, both in one and two dimensions, to grids of squares (Figures 4(a) and 4(b)) to provide a better appreciation of the differences and similarities between these techniques. In order to simplify the comparison of these techniques, the grids are "normalized" to the same-sized display area before each distortion technique is applied. Further, system parameters are chosen so that similar magnification factors are applied in the central focus region.

2.1 Polyfocal Display [Kadmon and Shlomi]

Kadmon and Shlomi [1978] proposed a polyfocal projection for the presentation of statistical data on cartographic maps. Although their concept was

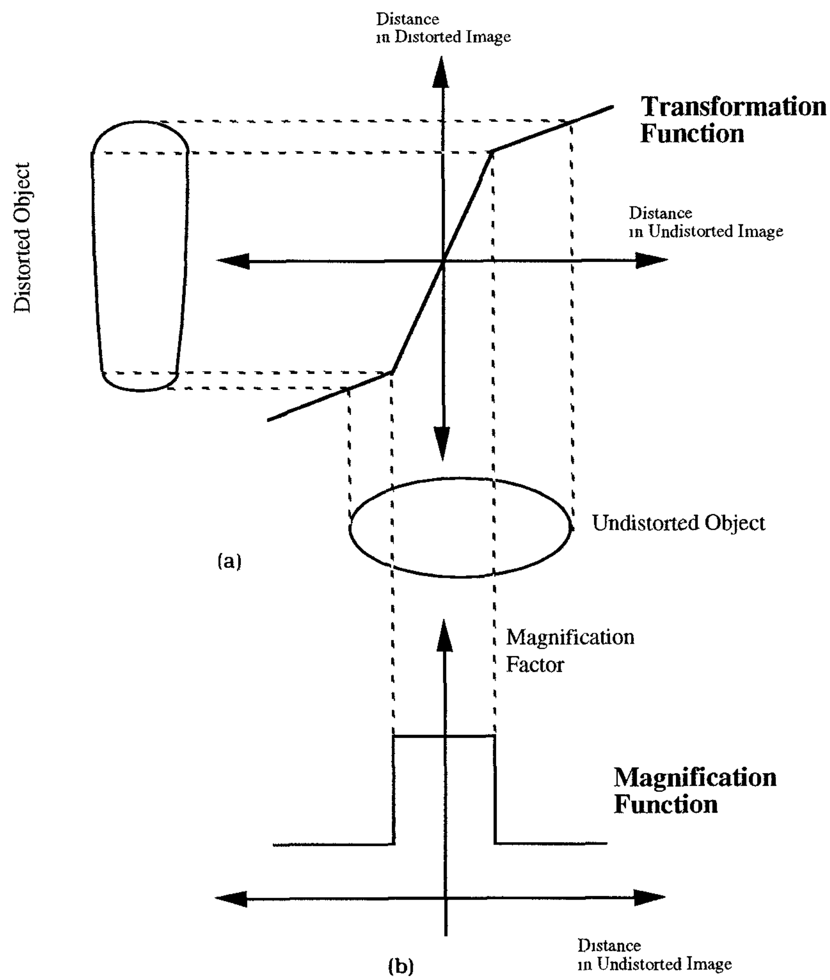

Fig. 3. (a) The transformation of an elliptic object by applying the transformation function of a Bifocal Display in one dimension; (b) the corresponding magnification function of the Bifocal Display.

applied in a noninteractive situation, they made a valuable contribution in laying down a solid mathematical foundation for many later distortion-oriented presentation techniques, although many of the later developments have been carried out without the knowledge of this work. Kadmon and Shlomi also proposed an implementation of a multifocal display. The graphical application of the Fisheye View [Sarkar and Brown 1992] could well be considered as a special case of the polyfocal projection.

The fundamental concept behind the polyfocal projection in its one-dimensional form can be illustrated by the transformation and magnification functions of Figures 5(a) and 5(b), where the highest peak (Figure 5(b)) is the focus of the display. (For a rigorous mathematical treatment of the polyfocal

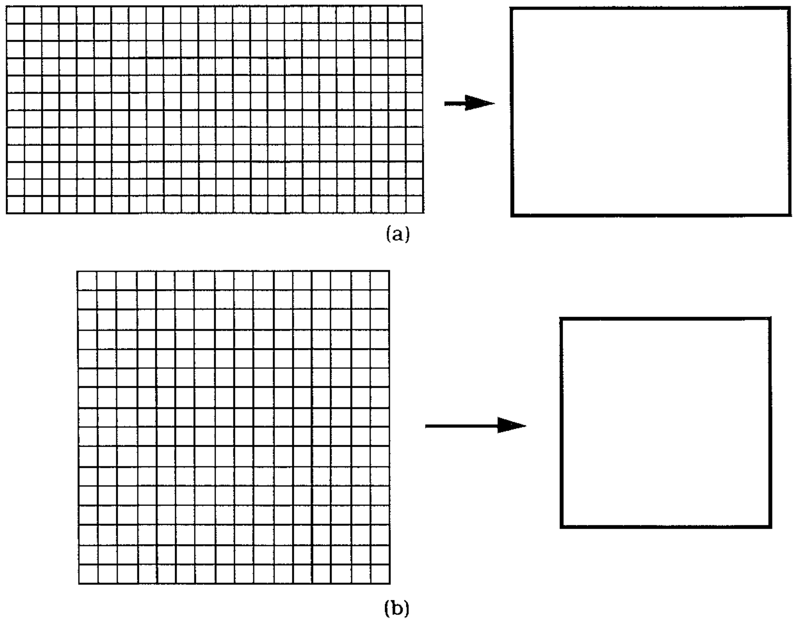

Fig. 4. A rectangular grid is to be mapped onto a confined space by applying a distortion-oriented technique; (a) in one dimension; (b) in two dimensions.

display, readers should refer to Kadmon and Shlomi's [1978] paper.) The curvature of the magnification function is controlled by two sets of parameters; one controls the magnification at the point of focus and the other the rate of change of magnification with distance from the point of focus. In cartographic terminology they are referred to as thematic variables. Figures 5(c) and 5(d) show the effects of this technique in one and two dimensions respectively. It should be noted that polyfocal projections distort the shape of the boundaries of the display. Further, the troughs in the magnification function, which are inherent in polyfocal projections, serve to compensate for the high magnification factors in the area surrounding the point of focus.

In the case of a multifocal polyfocal projection, there will be multiple peaks in the magnification function, each contributing a certain amount of "pull" to the entire image. In theory there is no restriction on the number of these "peaks" in the magnification function; the only limitation is the computation time involved and the comprehensibility of the resulting distorted image. Figures 5(c) and 5(f) show two displays with multiple foci; the former with the same parameters applied to each focus and the latter with different sets of values for each focus. It should be noted that it is possible to have zero magnification where a section of the display is effectively shrunk to nothing, thus creating a "vanishing area." Negative magnification factors may also be possible, creating overlapping views.

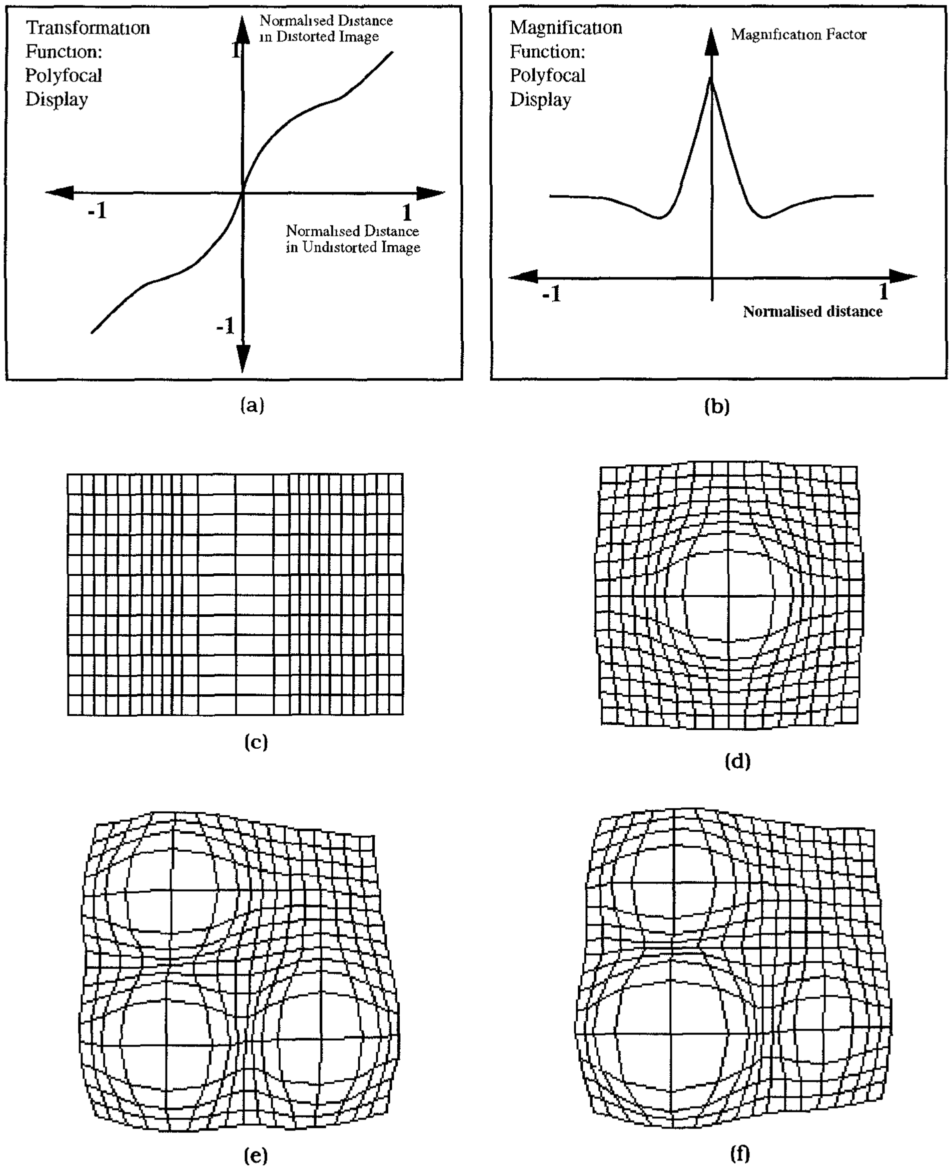

Fig. 5. The polyfocal projection: (a) a typical transformation function of a polyfocal projection; (b) the corresponding magnification function; (c) the application of the projection in one dimension; (d) the application of the projection in two dimensions; (e) a multiple-foci view of the projection using the same parameters for each focus point; (f) a multiple-foci view using different parameters.

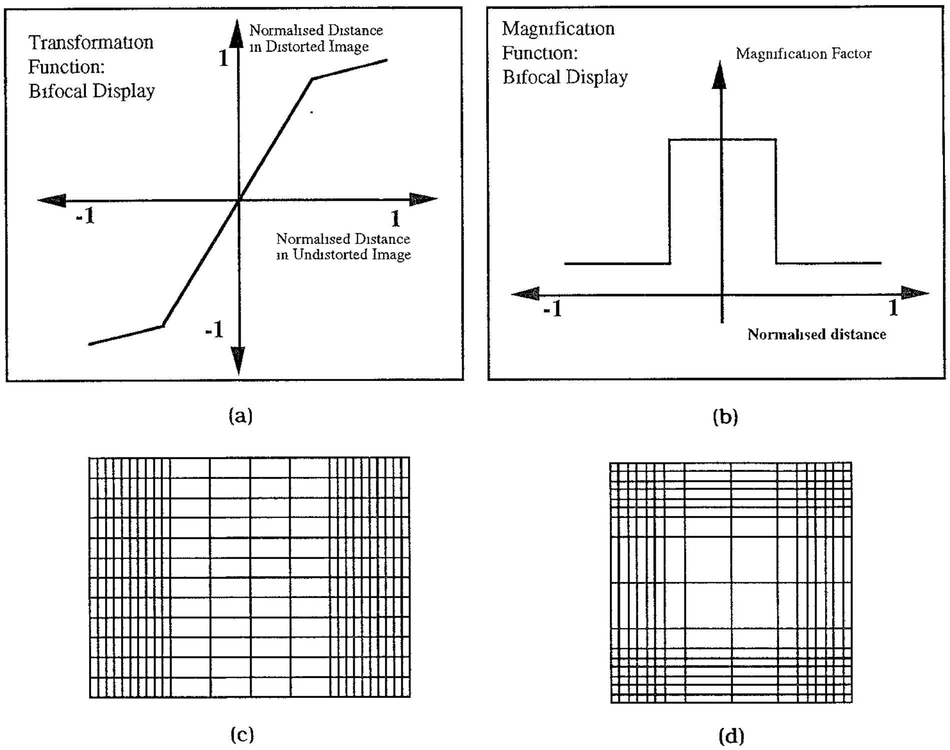

Fig. 6. The Bifocal Display: (a) a typical transformation function; (b) the corresponding magnification function; (c) the application of the display in one dimension; (d) the application of the display in two dimensions.

2.2 Bifocal Display [Spence and Apperley]

The Bifocal Display [Spence and Apperley 1982] in a one-dimensional form involves a combination of a detailed view and two distorted sideviews, where items on either side of the detailed view are compressed uniformly in the horizontal direction. Spence and Apperley used the mechanical analogy already referred to in Figures 2(a) and 2(b) to describe the display. The transformation and magnification functions for this technique are shown in Figures 6(a) and 6(b). Figure 6(c) shows a one-dimensional Bifocal Display applied to a square grid. Although the Bifocal Display is relatively simple in terms of implementation and does provide spatial continuity between regions, it has the disadvantage of discontinuity of magnification at the boundary between the detailed view and the distorted view. An analysis of the implementation requirements of the Bifocal Display based on special-purpose memory management hardware has also been described [Apperley et al. 1982].

Leung [1989] extended the bifocal concept to a two-dimensional form in an implementation of the London Underground map. Figure 6(d) shows the

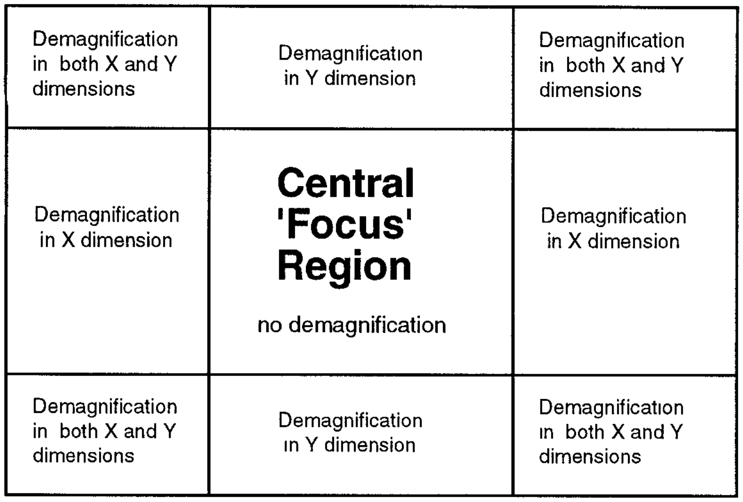

Fig. 7. Implementation of a 2D Bifocal Display.

effects of this technique in two dimensions. The visual area is subdivided into nine regions with a central focus region (see Figure 7), and other eight regions which are demagnified according to the physical position with respect to the central focus region; the same demagnification factor is used in both x and y directions in these regions. It should be noted that because the four corner regions are demagnified in both x and y directions using the same scale, these areas are not distorted. They are merely reduced in size.

2.3 Fisheye View [Furnas]

The Fisheye View concept was originally proposed by Furnas [1986] as a presentation strategy for information having a hierarchical structure. The essence of this technique is called *thresholding*. Each information element in a hierarchical structure is assigned a number based on its relevance (*a priori importance* or API) and a second number based on the distance between the information element under consideration and the point of focus in the structure. A threshold value is then selected and compared with a function of these two numbers to determine what information is to be presented or suppressed. Consequently, the more relevant information will be presented in great detail, and the less relevant information presented as an abstraction, based on a threshold value. Furnas' Fisheye View was illustrated by two text-based applications, one involving a large section of program code and the other a calendar. Koike [1994] considers the potential problem of this technique for presenting trees with different number of branches and offers an interesting refinement using fractal algorithms.

Mathematically the *degree of interest* (*DOI*) function, which determines for each point in the hierarchical information structure how interested the user

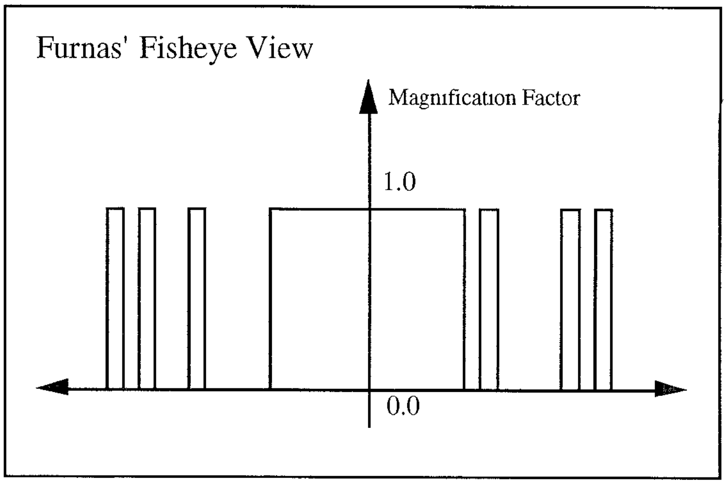

Fig. 8. A typical magnification function for Furnas' "Fisheye View"

is in seeing that point with respect to the current point of focus, is given by,

$$\mathbf{DOI}_{fisheye}(\mathbf{a}|. = \mathbf{b}) = \mathbf{API(a)} - \mathbf{D(a,b)},$$

where

(1) $\mathbf{DOI}_{fisheye}(\mathbf{a}|. = \mathbf{b})$ is the degree of interest in **a**, given that the current point of focus is **b**.

(2) **API(a)** is a static global value called *a priori importance* at point **a**; API values are preassigned to each point in the structure under consideration, and

(3) **D(a, b)** is the distance between point **a** and the point of focus **b**.

It is apparent that the **DOI** function of the Fisheye View is an information suppression function. The illustrations which Furnas used were text-based examples, and rather than involving demagnification per se, they involved the selective suppression and highlighting of components of the text depending on the prior degree of interest (DOI) values with respect to the object at the focus and a threshold value. The analogy with a traditional fisheye lens is cryptic. Furnas' technique can be described best by a magnification function, as shown in Figure 8.

A number of other implementations, all claiming to use this technique, have been reported and have created some confusion as to what a "fisheye view" really means. These implementations are not only different in their application domains, but also in their form, and they will be examined in detail in later sections.

2.4 Fisheye View [Hollands et al.]

Hollands et al. [1989] represented a fictitious subway network using both a Fisheye View and a simple scrolling view, and compared the users' performance with these two interfaces. Users performed three different tasks: a route task, a locate/route task, and an itinerary task. Although no details were provided of the implementation of the Fisheye View, the figures in their paper suggest that it is a graphical implementation of a much more general fisheye concept, which has more in common with the Bifocal Display than with Furnas' DOI functions. Furthermore, the station symbols displayed in the focus region of the Fisheye View were smaller than those in the scrolling view, apparently contradicting the fundamental concept of *degree of interest* [Furnas 1986]. The transformation and magnification functions used would appear to be similar to those of the Bifocal Display (Figures 6(a) and 6(b)).

2.5 Fisheye View [Mitta]

Mitta [1990] proposed a "fisheye" strategy for the presentation of aircraft maintenance data. The example used showed a solenoid assembly consisting of a number of components presented in different views. In each of these Fisheye Views certain components were suppressed so that users could focus their attention on the parts which were presented on the display screen. In the conclusion of the paper, Mitta wrote "Thus, future research efforts are to examine how information should be selected, in addition to what information should be presented" confirming that the technique used was an information suppression technique rather than the more conventional notion of a Fisheye View used by Hollands et al. [1989].

Mitta made reference to Furnas' work on Fisheye Views and extended a multiple-focus-point version of the same technique.

2.6 Perspective Wall [Mackinlay et al.]

The Perspective Wall [Mackinlay et al. 1991], a conceptual descendent of the Bifocal Display, is based on the notion of smoothly integrating detailed and contextual views to assist in the visualization of linear information.

The principle behind the Perspective Wall is illustrated in Figures 9(a) and 9(b). The two side panels, which show a distorted view of the out-of-focus regions, are demagnified directly proportional to their distance from the viewer; the corresponding transformation and magnification functions for this technique are shown in Figures 10(a) and 10(b). Although this technique is inherently two dimensional, for illustrative purposes its application to the two square grids in both one and two dimensions is shown in Figures 10(c) and 10(d). The main distinction between this technique and the Bifocal Display is that in the out-of-focus regions, the Perspective Wall demagnifies at an increasing rate in comparison with the Bifocal's constant demagnification (compare the magnification functions in Figures 6(b) and 10(b)). This rate of increase in the magnification function of the two side panels depends on the angle Θ; the greater this angle, the flatter the slope. There is a discontinuity in the magnification function at the points where the two side panels meet the middle panel; the bigger the angle Θ, the greater the discontinuity.

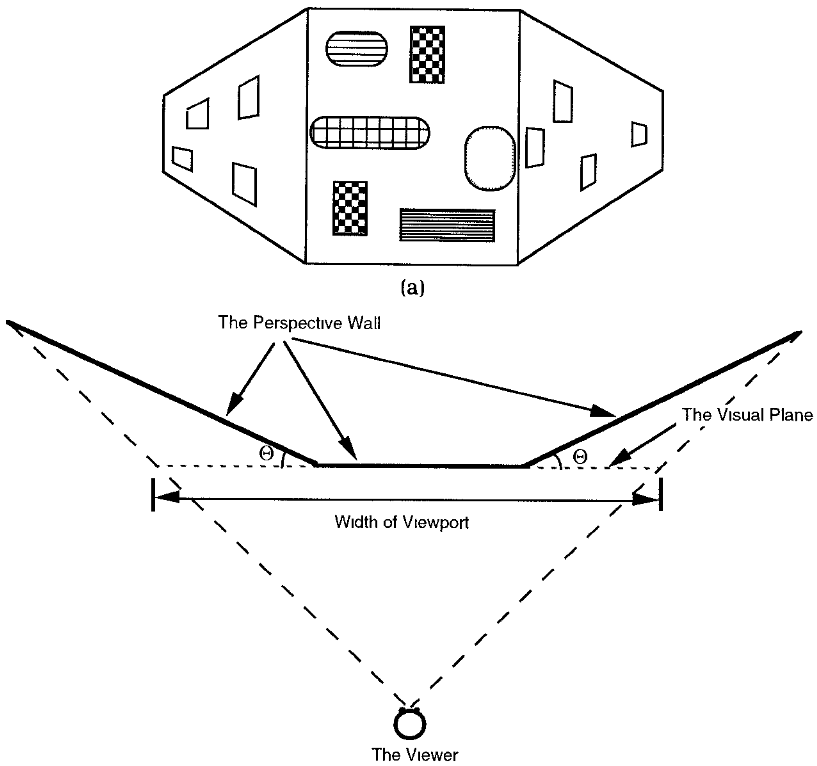

Fig. 9. With two side panels positioned at an angle the Perspective Wall provides a distorted view to the viewer; (b) a plan view of the Perspective Wall showing the relationships between the wall, the viewport, and the viewer.

The view generated by the Perspective Wall is dependent on a number of parameters: the length of the wall, the width of the viewport, the angle Θ, the size of the central region, etc. To get a better understanding of the Perspective Wall, consider the effect of increasing the angle Θ (Figure 9) while all other system parameters remain constant. As the angle Θ increases with the two side panels tilting backward (see Figure 9(b)), as a consequence, the viewer will have to be positioned further away from the wall because the width of the viewport is fixed. It should be noted that the position of the viewer determines the projection of the two side panels on the visual plane (see mathematical derivation of the transformation function in the Appendix). As the angle Θ is increased further, there is a position where the viewer is essentially positioned at infinity. At this point the demagnification in the peripheral regions will be constant, and it can be seen that the Bifocal Display is actually a special case of the Perspective Wall. This point can be seen also with the mechanical analogy of Figures 2(a) and 2(b); a close-up

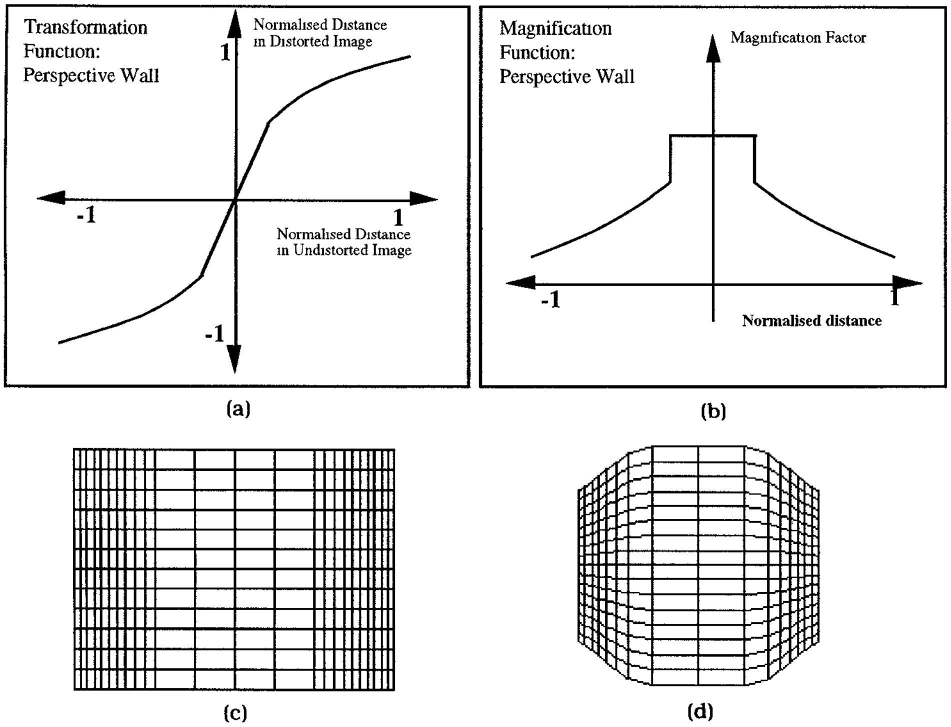

Fig. 10. The Perspective Wall: (a) a typical transformation function; (b) the corresponding magnification function; (c) the application of the wall in one dimension; (d) the application of the wall in two dimensions. Here the number of dimensions relates to the dimensions in which the perspective transformation is applied on the projection, not to the dimensionality of the model on which the projection is based.

view would produce a Perspective Wall, and a view from infinity, a Bifocal Display.

The Perspective Wall does add a full 3D feel to the otherwise flat form of the Bifocal Display. However, this effect is produced at the cost of wasting expensive "real estate" in the corner areas of the screen, contrary to one of the prime objectives of distortion techniques to maximize the utilization of the available display area. This particular shortcoming of the Perspective Wall has been overcome more recently with the development of the Document Lens technique [Robertson and Mackinlay 1993].

2.7 Graphical Fisheye Views [Sarkar and Brown]

Sarkar and Brown [1992] extended Furnas' fisheye concept and laid down the mathematical formalism for graphical applications of this technique. They proposed two implementations, both of topological networks, one based on a Cartesian coordinate transformation system and the other on a polar system. Owing to the nature of polar transformation, in theory a straight line and

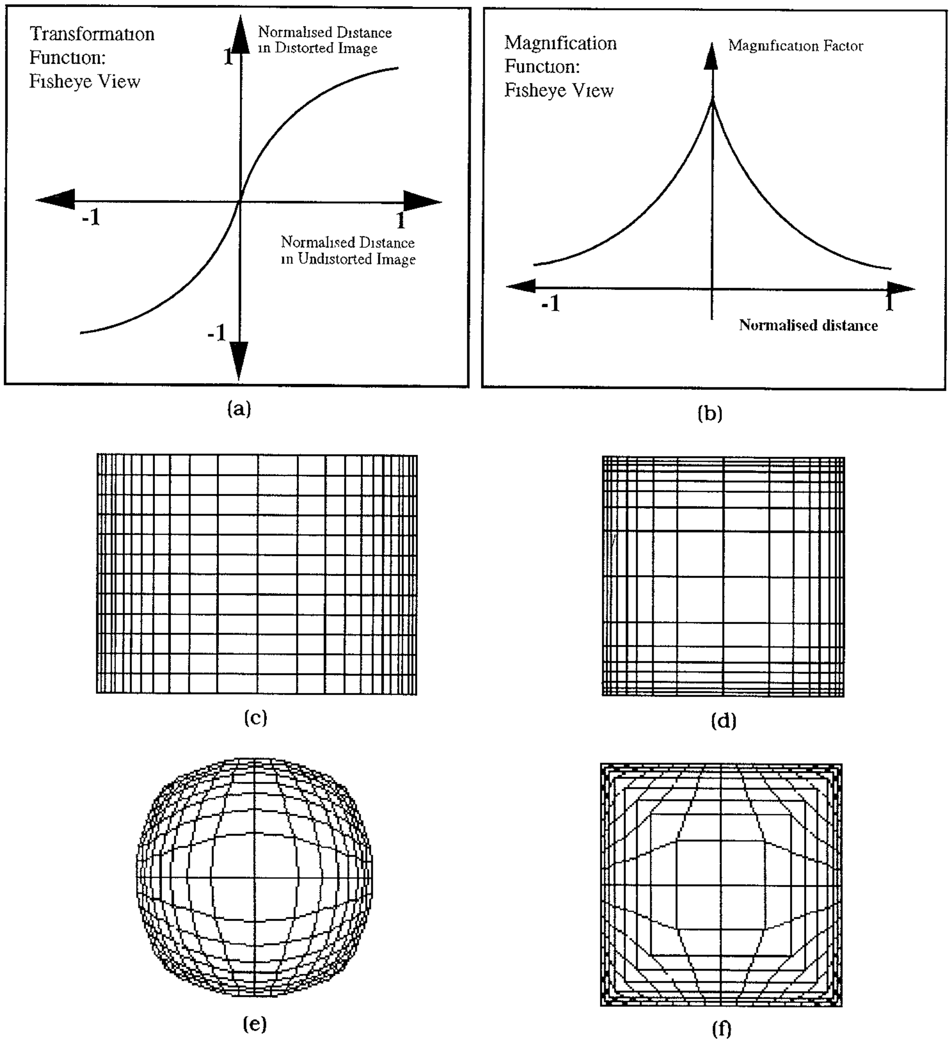

Fig. 11. The Fisheye View: (a) a typical transformation function; (b) the corresponding magnification function; (c) the application of the Fisheye View in one dimension; (d) a Cartesian Fisheye View in two dimensions; (e) a polar Fisheye View; (f) a normalized polar Fisheye View.

rectangle will normally be transformed into a curved line and a curvilinear rectangle respectively. To overcome this problem, the transformation was applied only to the nodes of the structure, and the nodes were then connected by straight lines. The transformation and magnification functions for the Fisheye View are respectively, (see Figures 11(a) and 11(b))

$$T(x) = \frac{(d+1)x}{(dx+1)} \quad \text{and} \quad M(x) = \frac{(d+1)}{(dx+1)^2},$$

where

—d is called the distortion factor; the larger this number is, the bigger the magnification and the amplitude of the peak in the magnification function; and,

—x is the normalized distance from a point under consideration to the point of focus. x can have a value $0 <= x <= 1$. If $x = 0$, the point under consideration is at the point of focus, and if $x = 1$, it is at a position furthest away from the point of focus on the boundary.

Figure 11(c) shows the application of this Fisheye View in one dimension. Figure 11(d) shows the two-dimensional Fisheye View with a Cartesian coordinate system, and Figure 11(e) with the transformation based on a polar coordinate system. It is interesting to note that the polar Fisheye View produces a rounded appearance which unfortunately does not provide a natural look when implemented on a rectangular screen. Sarkar and Brown proposed further that the rounded appearance of the Polar Fisheye View be remapped on a rectangular space; the result of this modified transformation is illustrated in Figure 11(f). Surprisingly, the appearance of Figure 11(f) bears some resemblance to that of a Perspective Wall (Figure 10(d)). As this perspective transformation is applied fully in two dimensions (the perspective transformation is not applied in the vertical direction in the middle panel for the Perspective Wall proposed by Mackinlay et al.), a more appropriate name for this technique would be Perspective Space [Leung and Apperley 1993a].

While these fisheye transformations provided the spatial distortion in two dimensions, Sarkar and Brown [1992] introduced a further information magnification in the third dimension based on the concept of *a priori importance* (API) proposed by Furnas. Their implementation of API was extended to three separate functions called $\text{Size}_{fisheye}$(S), Visual Worth (VW), and $\text{Details}_{fisheye}$(DTL). The purpose of these functions is twofold: first, they provide a flexible information suppression/enhancement mechanism to generate an effective Fisheye View, and second, the resulting display provides the viewer with a three-dimensional feel. This technique is potentially very powerful in displaying information which is multilayered and globally organized in a hierarchical tree or network structure.

Misue and Sugiyama [1991] described two transformation functions (polar and Cartesian versions) for graphical Fisheye Views which have some similar properties to those of Sarkar and Brown.

3. A TAXONOMY OF DISTORTION-ORIENTED PRESENTATION TECHNIQUES

An examination of the transformation and magnification functions of the distortion-oriented presentation techniques described in the previous section (see Figure 12) reveals their underlying differences and similarities. These techniques can be classified conveniently in terms of their magnification functions: basically, there are two distinct classes. One class of these techniques has piecewise continuous magnification functions; the Bifocal Display

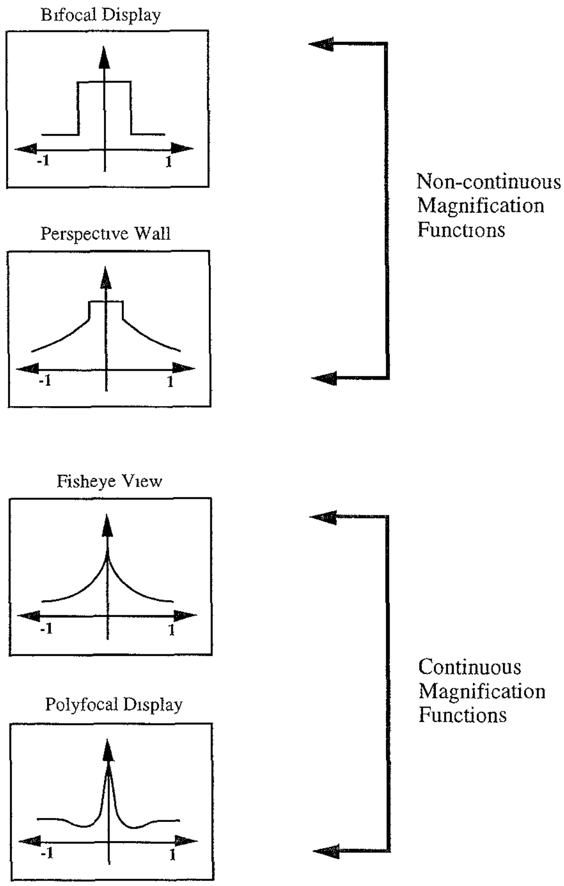

Fig. 12. A taxonomy of distortion-oriented presentation techniques.

and the Perspective Wall are typical examples. The other class has continuous magnification functions; the Fisheye View and the Polyfocal Projection belong to this second class.

Techniques with piecewise continuous functions can be classified further into those with constant or varying magnification functions; the Bifocal Display belongs to the former subclass and the Perspective Wall the latter. As explained in Section 2.6, the Bifocal Display is a special case of the Perspective Wall. A display which has multiple discrete levels of magnification in the magnification function could be generated; the limitation of extending the Bifocal Display concept to a higher level is imposed only by the system's resources. Further, the magnification factors used in these levels may be chosen in such a way that the function approximates to a continuous one. Figure 13 shows the general layout of a display with three magnification levels, and Figure 14 shows the magnification function for a display with four magnification levels which approximates that of a Fisheye View. Applica-

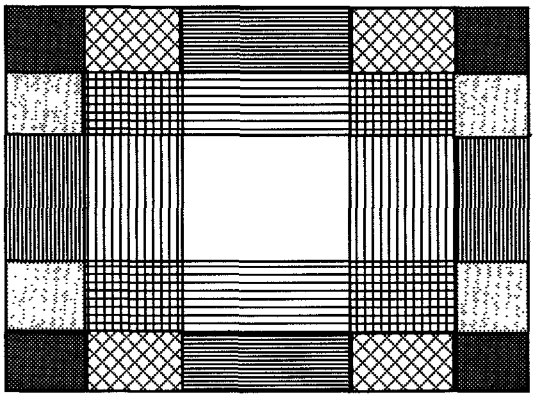

Fig. 13. The 25 regions that would be generated by extending a 2D Bifocal Display to incorporate three distinct magnification levels, rather than two.

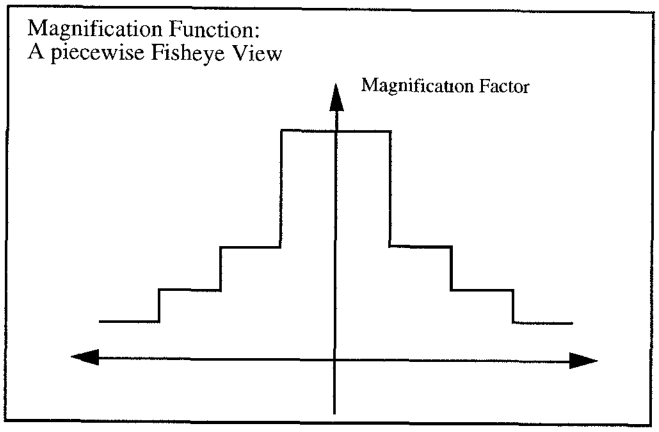

Fig. 14. The magnification function of a piecewise Fisheye View.

tions, and the complexity involved in the implementation, of these techniques are discussed in later sections.

Techniques with continuous magnification functions have one undesirable attribute; they tend to distort the boundaries of the transformed image. The bigger the magnification factor at the focus is, the bigger this distortion at the boundaries will be. This is because these techniques are generally applied radially rather than independently in the x and y directions. Consequently, the corner areas are pulled in toward the point of focus. This problem can be overcome in two ways, as implemented by Sarkar and Brown [1992] in their Cartesian and Polar Fisheye Views. First, the transformation may be applied independently in the x and y directions as in the Cartesian Fisheye View (Figure 11(d)). Second, the distorted boundaries can be remapped onto a rectangular size of the display area as illustrated in Sarkar and Brown's

Polar Fisheye View (Figure 11(e)). It should be noted that because of the irregular shape of the boundaries in the Polyfocal Projection which is inherent in its transformation, more extensive calculation would be required in this case to perform the remapping operation.

A closer examination of the magnification functions for the Fisheye View and Polyfocal Projection (Figure 12) shows their strong similarities in their general profiles. One could consider the Fisheye View as a special case of Polyfocal Projection. The difference in these two functions is the dips in Polyfocal Projection's magnification function. It is the dips in the Polyfocal Projection's magnification function which make it possible for this technique to support a multiple-focus presentation as shown in Figures 5(e) and 5(f); techniques which do not have this property in their magnification function will not be able to provide a flexible multiple-focus system. This point is discussed further in a later section on implementation issues (Section 5.2).

4. A UNIFIED THEORY

While the taxonomy in the previous section gives a global view of distortion-oriented techniques, a unified theory is proposed here to provide a better insight and understanding of their underlying concept.

The simplest way of visualizing the working of a distortion-oriented presentation technique is to treat the displayed information as if it was printed on a stretchable rubber sheet mounted on a rigid frame.[1] This is an effective analogy which has been used by various researchers to describe distorted displays [Tobler 1973; Mackinlay et al. 1991; Sarkar et al. 1993]. The rubber sheet is densely populated with information to the extent that in its unstretched form, the viewer can see only the global context of the information structure and is not able to make out any detailed information from it. In order that a viewer can examine a particular section to access detailed information, the rubber sheet has to be stretched. Any stretching of the rubber sheet is analogous to applying magnification to a section of the screen. As the rubber sheet is mounted on a rigid frame, any stretching in one part of the sheet results in an equivalent amount of "shrinkage" in other areas. The consequence of this stretching and shrinking of the sheet is an overall distorted view. The amount of stretching or magnification and the manner in which it is applied on the sheet depend entirely on the magnification function of the distortion technique used.

To illustrate how this theory works, consider that the Bifocal Display technique is to be applied on a rubber sheet mounted on a rigid frame as shown in Figure 15(a). Three points, **a**, **b**, and **c** are marked on the sheet to show the effect of stretching. The dotted lines enclose an area in the middle to

[1] It will be necessary for the edges of the sheet to be able to slide along the edges of the frame.

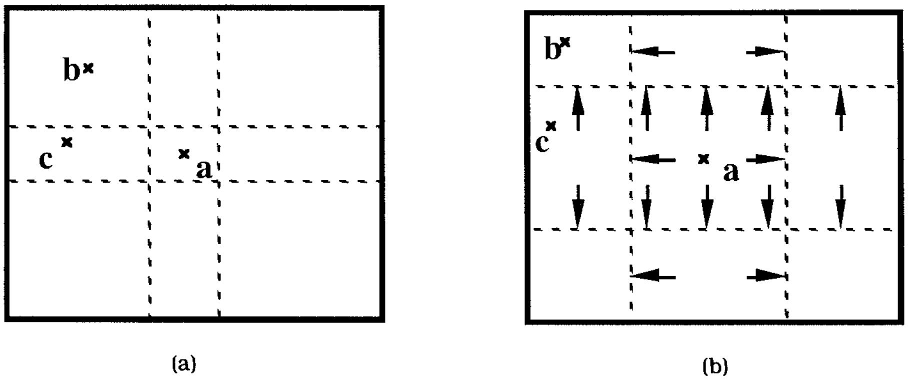

Fig. 15. (a) An unstretched rubber sheet mounted on a rigid frame and the positions on it of three points **a**, **b**, and **c**. Stretching is to be applied at the dotted lines; (b) the arrows indicate the directions of stretching applied to the sheet. Point **a** is not displaced since it is at the focus. Points **b** are **c** are both displaced.

be magnified in order that the viewer can examine its contents in detail; forces are applied along these lines to provide the magnification effect.

Figure 15(b) shows the sheet after stretching is applied in the directions of the arrows. As point **a** is located exactly at the point of focus and all the forces balance out, no displacement results at point **a**. Point **b** experiences two orthogonal forces as a consequence of the stretching applied near the top left-hand corner area. The stretching in these two directions causes **b** to be displaced in both directions toward the top left-hand corner. As a result, the four corner areas are being shrunk by an equivalent amount to accommodate the excess area caused by the stretching. Point **c** experiences three forces, two stretching forces applied vertically and a compressing force horizontally. If point **c** were situated at the midpoint between the two dotted lines, no vertical displacement would take place; in this case because **c** is situated above this midpoint, the resultant force displaces point **c** upward. At the same time, the compressing force that point **c** experiences causes shrinkage in the horizontal direction as indicated in Figure 15(b).

In the case of a multiple-focus view, the situation is similar. The only difference is that stretching or magnification will occur in a greater number of areas on the rubber sheet. The important fact is that the sum of all stretchings or magnifications must be equivalent to the total shrinkages or demagnifications. Otherwise, the rigid frame holding the rubber sheet would deform either because of insufficient surface to accommodate the "over-stretched" sheet or because of an oversupply of space to fit an "overshrunk" sheet. The former situation applies to the Polyfocal Projection (Figures 5(d)–5(f)) while the Polar Fisheye View (Figure 11(e)) and the Perspective Wall (Figure 10(d)) are examples of the latter. As explained in the previous section, techniques with continuous magnification functions by their mathematical nature deform the rectangular frame because of the radial influence inherent in the transformation. "The unity gain at the periphery insures continuity retention in the interface to the real world" [Farrand 1973, p. 32].

5. DISCUSSION

5.1 Performance Issues

Although the techniques discussed in this article may be used to display static distorted images on the computer screen, in the context of human-computer interaction an input device will be used to support real-time interaction by users. To allow presentation and navigation of an information space, there are generally three basic interaction methods to effect a change of viewport using an input device: scrolling, pointing and selecting, and dragging.

With scrolling, as the user initiates a movement with the input device (e.g., moving a finger on a touch-sensitive screen or scrolling a mouse), the system detects the direction of the movement and updates the image on the display screen in real time; the amount of movement effected on the central focus area is directly proportional to the scrolling action on the input device made by the user. Depending on system response time, the implementation of scrolling usually involves the creation and the display of a number of intermediate images between the source image to the target image to provide a smooth, continuous visual transition as the focus region is repositioned. To improve performance, detail can be omitted from the nonfocus areas during interaction [Robertson and Mackinlay 1993].

With pointing and selecting, the user moves the central focus region to another location by first positioning the cursor using the input device, and then activating it to select the desired point of interest. The new display with a change of the focus region and its surrounding areas will be presented then.

Dragging incorporates features of both the previous methods. The user selects an item of interest and at the same time moves it (typically by a concurrent scrolling action) with an input device to a position desired by the user for detailed examination. To maintain context with this form of interaction, usually it will be necessary to have the central focus region fixed with respect to the display surface, with the data space appearing to move underneath. This will necessarily result in some regions of the display not being fully utilized if the point of interest is near a corner of the space, and in some areas of the space either not being shown, or being severely distorted.

Distortion-oriented techniques are inherently complicated in their implementation, and some require a significant amount of system time to generate a new image. While an excessively long system response time would render an interface "unusable," this problem may be overcome by using dedicated computer hardware and memory management systems to support the implementation of such techniques [Apperley et al. 1982; Card et al. 1991]. Further, as general-purpose graphics hardware becomes increasingly sophisticated and powerful, effective software solutions have become practicable [Robertson and Mackinlay 1993]. Also, it should be noted that a system response time that is too fast could be just as disconcerting to the user. The sudden shift of a distorted view or any fast scrolling movement on the display screen could cause visual discomfort to the viewer over prolonged, continuous use. This effect is similar to watching a home video taken by an amateur who panned the view jerkily at high speed.

Although there has been an increasing amount of research carried out on user performance in reading moving text on computer displays [Kang and Muter 1989; Chen and Chan 1990], little work has been done to investigate the effects of moving graphical images or to find out the optimum speed for scrolling graphical images on computer screens. Before empirical findings in this research area are available, systems with too short a response time will have to be slowed down by introducing delays during image updates on a trial-and-error basis. Fortunately, this problem relates only to high-performance computer systems, and generally it is easier to slow a system down than to speed it up.

5.2 Implementation Issues

The selection of an interface and its implementation are dictated often by the system hardware available, and its computational power. The complexity of a presentation technique will, therefore, have much influence on this decision. Although distortion-oriented techniques tend to be complex in their implementation, their complexities differ quite widely and depend primarily on the mathematical transformation functions used. Furthermore, very often, tradeoffs between the computational power of the hardware and system memory can be made to yield optimum implementation. For example, distorted displays based on stepwise magnification functions may have their different views created and stored in memory in advance. The generation of a distorted view in real time will involve only the cutting and pasting of various sections of these bit maps stored in memory. Generally, systems with less computational power perform the operation of shifting graphic bit maps much faster than that of carrying out complicated mathematical calculations in real time. However, such systems do require adequate on-board memory to support the interface for satisfactory performance. In an implementation of the London Underground map using the Bifocal Display technique [Leung 1989], four separate bit maps, each with different magnifications applied in x and y directions, are stored in memory; altogether, the four bit maps take up six megabytes of system memory. As the user scrolls the mouse, the Bifocal Display is generated by cutting and pasting various sections of these bit maps in real time to generate the nine regions as shown in Figure 7. A similar technique has been applied in implementing the stepwise magnification function of the document lens, where the text is rendered for each of the five regions of a truncated pyramid in advance, and then clipped, scaled, and translated as appropriate during interaction [Robertson and Mackinlay 1993].

Display techniques using a continuous magnification function pose a problem for this implementation method. This is because of the continuum of magnification factors the system will have to cater to at every possible position of the point of focus on the image; the number of bit maps that have

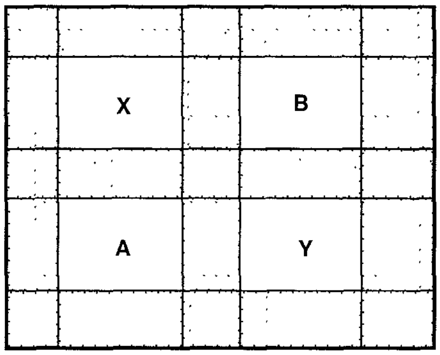

Fig. 16. A common problem with multifocus presentations. Intended focus areas are **A** and **B**. Unintentional focus areas **X** and **Y** are created.

to be stored will be too large to be practical for implementation. One way of overcoming this problem is to use a piecewise continuous magnification function to approximate a continuous function (see Figure 14). This method is an extension of the Bifocal Display to multiple magnification factors with a stepwise function.[2] It can be shown that if the number of distinct magnification levels is n, the number of bit maps the system will have to maintain is n^2. For example, Figure 13 shows a two-dimensional Bifocal Display extended to have three distinct levels of magnification; there are 25 regions on the screen and nine distinct mappings of the data to the display.

Interfaces with a scrolling-style interaction use this multiple-bitmap method typically to generate the distorted view and therefore require less computational power but demand greater system memory. In contrast, interfaces with dragging and pointing and selecting inputs will rely on the computational power of the system to generate the images by performing the mathematical transformation in real time. Dedicated hardware to support the interface may be considered for implementation if a piecewise approximation of the transformation function is not desirable. It is interesting to note that although the Perspective Wall has a piecewise continuous magnification function, the mathematical transformation for the two side panels involves fairly complicated calculations.

Multiple-focus views, which are akin to a multiple-window environment in some text-based and graphical systems, are often desirable. For example, if the user wishes to examine two entities that are located at the extremes of the display, a multiple-focus view would facilitate this application. However, there are some inherent conceptual limitations with the Cartesian (independent x and y) techniques in implementing multiple-focus views. To illustrate this point, consider the case where two focus views A and B are to be created on a Bifocal Display as shown in Figure 16. Because of the inflexibility in the transformation function, two unintended focus views are created at x and y as a side effect. The inflexibility applies typically to techniques whose magni-

[2]It would be tempting to refer to this as a trifocal, quadrafocal, etc. display. However, because of the use of the term polyfocal display to refer to a display with multiple foci, rather than multiple magnification factors, this terminology has been avoided.

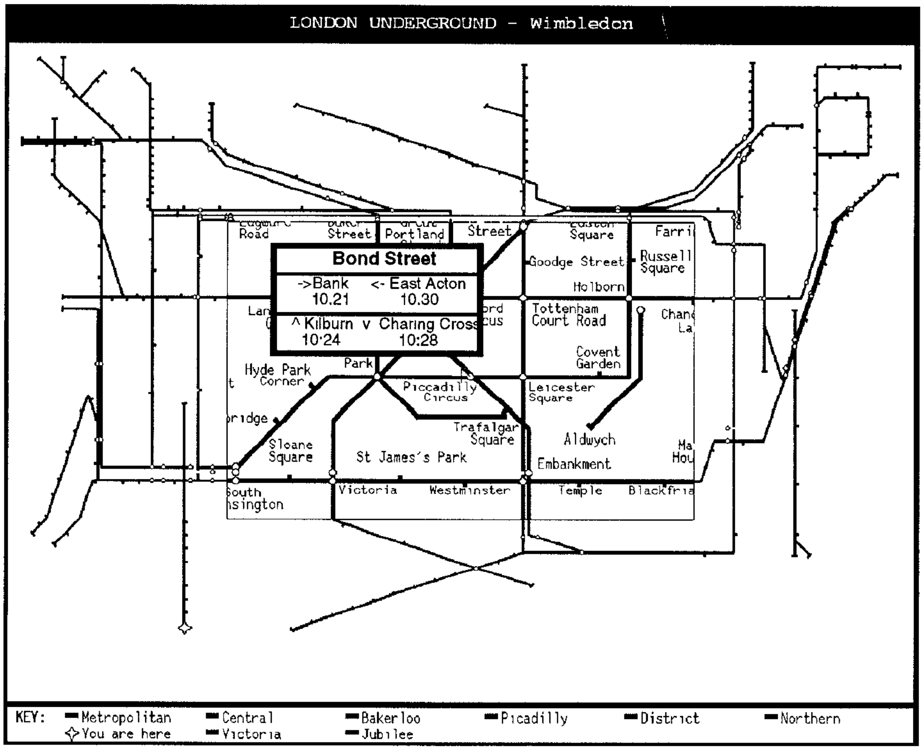

Fig. 17. An application of the combined spatial and information enhancement technique using a Bifocal Display. The train departure time information for Bond Street station, which is embedded in the station symbol, is revealed by user activation.

fication functions do not have a dip in them like that of the Polyfocal Projection (Figure 6(b)). One way of alleviating this problem is to facilitate a pop-up-window-type arrangement to support multiple views. However, this may create additional navigational problems for the user because of the discontinuity of the presentation in the detailed and demagnified views on the display.

5.3 Hybrid Techniques and Application Domains

Although problems associated with presenting large volumes of data in a confined display screen area may be classified into spatial problems or information density problems, there are applications where both issues are relevant.

Consider a computer-based information system which provides information to the user about the time of arrival of the next train at any station on the London Underground map. Such a system entails two separate presentation problems. First, the London Underground map needs to be presented to the user to facilitate easy navigation. Second, the information about arrival times needs to be embedded in the map to avoid information clutter. Figure 17 illustrates an effective solution to this combined problem. In this example, the Bifocal Display technique has been used to tackle the spatial presentation problem; the user can navigate freely on the London Underground map, examining a small area in detail while maintaining global context of the map. When the user has located the station of interest, in this case Bond Street station, the embedded information is then revealed. This technique is potentially powerful, and greater research effort should be focused on exploring the application domains for such hybrid approaches.

Distortion-oriented techniques are very useful in solving the spatial problem. However, they should be used with some caution. Due consideration should be given to the type of information to be conveyed and how it will be perceived by the user. For example, in applications where the information to be presented is not well structured, these techniques may not have the desired effect. It should be pointed out that the Polyfocal Projection was originally intended for thematic cartography where maps are presented with a specific theme such as population density or temperature, rather than to show the absolute spatial distances between cities or countries. Leung and Apperley [1993b] discuss the relationship between these presentation techniques, the nature of the original data and its graphical representation, the physical characteristics of the display system (including resolution), the style of interaction, and the task being carried out.

6. CONCLUSION

Generally, there are two problems associated with the presentation of data in a confined space: a spatial problem and an information density problem. The Fisheye View concept that was first proposed by Furnas and later extended by Mitta is an information suppression technique aimed at solving the latter. In this context, the suppression of information creates an "information distortion." Such techniques are very different to those applied to spatial problems as discussed in this article.

This article has presented a taxonomy and a unified theory of graphical distortion-oriented presentation techniques for spatial problems. Depending on the problem domain, these techniques may be applied in both one or two dimensions. Based on their magnification functions, distortion-oriented techniques may be classified into two categories: those with continuous functions and those with noncontinuous functions. The Bifocal Display and the Perspective Wall belong to the former class, and the Polyfocal Projection and the Fisheye View to the latter. From an implementation viewpoint, multiple-focus regions are practical only with the Polyfocal Projection because other distortion-oriented techniques create extra unintended focus regions as a side effect.

The formalism put forward by Sarkar and Brown on the Fisheye View has laid down the ground work for graphical application of this technique for spatial problems. However, a number of variations of the implementation of this technique are possible.

The unified theory presented in this article has shown how magnification and demagnification work in tandem to create the desired distorted view. There is really no limitation on how these distorted views could be generated. A simple way of explaining these distortion techniques is to treat the display surface as a stretchable sheet of rubber mounted on a rigid rectangular frame. Magnification or "stretching" is carried out based on some mathematical transformation operating within that space. The basic law governing distortion-oriented techniques, which is a corollary of Newton's third law of motion, simply states that "where there is a magnification, there will be an equal amount of demagnification to compensate for the loss of display area in a confined space; otherwise the area of that confined space will change."

This article has aimed to demystify the complex mathematics and clarify the unnecessary confusion caused by different terminologies used in current literature. Research efforts should now be focused on a number of interrelated areas. First, a better understanding of these distortion techniques from the HCI perspective should be aimed at by gathering empirical evidence to evaluate the usability of these interfaces. Evaluation of graphical user interfaces is a highly complex task, and a multidimensional approach [Burger and Apperley 1991] is recommended because it provides a comprehensive view for effective interface evaluation. Second, with a better understanding of the usability of these techniques, optimum application domains can then be identified. Third, algorithms or specific hardware architectures should be developed to optimize system response time to enable these techniques to be applied in complex real-time situations. Finally, other nondistortion techniques, such as information suppression, should be investigated further since they are potentially powerful. They could be applied concurrently with the distortion-oriented techniques discussed in this article to complement their effectiveness.

APPENDIX

This section presents the mathematical derivation of the transformation and magnification functions for various distortion-oriented presentation techniques discussed in this article. (See Table A.I for variables and notations.) The transformation function of a distortion-oriented technique defines the way in which a point in the original object image is transformed to the distorted target image, and the magnification function describes the degree of distortion which has been applied to a particular point of interest. Mathematically these two functions are related; the magnification function is the first-order derivative of the transformation function.

Because of the symmetrical nature of two these functions, only the positive horizontal x dimension of the object image has been used for their derivation. For points on the negative horizontal axis, the following relationships apply:

$$\text{Transformation Functions} \quad T(-a) = -T(a)$$
$$\text{Magnification Functions} \quad M(-a) = M(a)$$

where a has a positive value.

Table A.I. Variable and Notation Used in the Appendix

Symbol	Meaning
A	One of the two constants used in Polyfocal Projection
a	The boundary point between two regions of the object image in Bifocal Display and Perspective Wall
b	The boundary point between two regions of the target image in Bifocal Display and Perspective Wall
C	One of the two constants used in Polyfocal Projection
d	Distortion factor used in Fisheye View
k	Equivalent to $\frac{(1-a)}{(1-b)}$, and used in the derivation of the magnification function of Perspective Wall
$M(x)$	Magnification function
$T(x)$	Transformation function
V_y	The distance between the viewer and the visual plane of the Perspective Wall
x	A point variable on the horizontal axis
θ	The angle between the Perspective Wall and the visual plane

A1. Polyfocal Projection [Kadmon and Shlomi 1978]

The transformation function of the Polyfocal Projection is given by

$$T_{polyfocal}(x) = x + \frac{A.x}{(1 + C.x^2)} \tag{1}$$

where A and C are constants.

The magnification function of the Polyfocal Projection is,

$$\begin{aligned} M_{polyfocal}(x) &= \frac{d}{dx}\left(T_{polyfocal}(x)\right) \\ &= 1 + \frac{A.(1 + C.x^2) - A.x.2.C.x}{(1 + C.x^2)^2} \\ &= 1 + \frac{A - A.C.x^2}{(1 + C.x^2)^2} \end{aligned}$$

$$M_{polyfocal}(x) = 1 + \frac{A.(1 - C.x^2)}{(1 + C.x^2)^2} \tag{2}$$

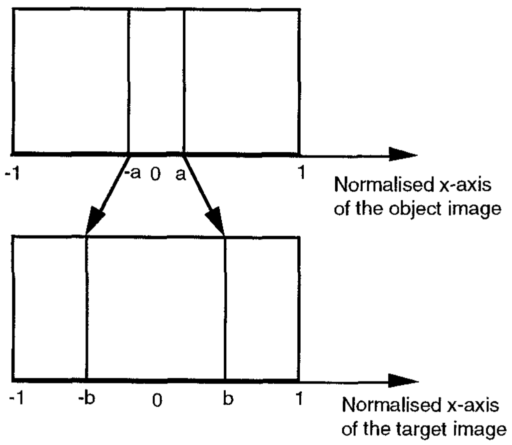

Fig. A.1. The relationships between the object image and target image.

A2. Bifocal Display [Spence and Apperley 1982]

From Figure A.1, the transformation functions of the Bifocal Display can be derived as follows:

$$\text{for } x \leq a, \quad T_{bifocal}(x) = x.\frac{b}{a} \tag{3}$$

$$\text{for } x > a, \quad T_{bifocal}(x) = b + (x - a).\frac{(1-b)}{(1-a)}. \tag{4}$$

The magnification function of the Bifocal Display is

$$M_{bifocal}(x) = \frac{d}{dx}\big(T_{bifocal}(x)\big),$$

$$\text{for } x \leq a, \quad M_{bifocal}(x) = \frac{b}{a} \tag{5}$$

$$\text{for } x > a, \quad M_{bifocal}(x) = \frac{(1-b)}{(1-a)}. \tag{6}$$

A3. Perspective Wall [Mackinlay et al. 1991]

Figure A.2 shows an elevated view of a Perspective Wall and the relationships between the object image and target image. Figure A.3 shows a simplified diagram of Figure A.2 with the coordinates of a number of key reference points used to derive the transformation function. In order to derive the transformation function of the Perspective Wall, the position of the viewers with respect to Wall will have to be determined first. The position of the viewer is dependent on the width of the viewport, the length of the side panel

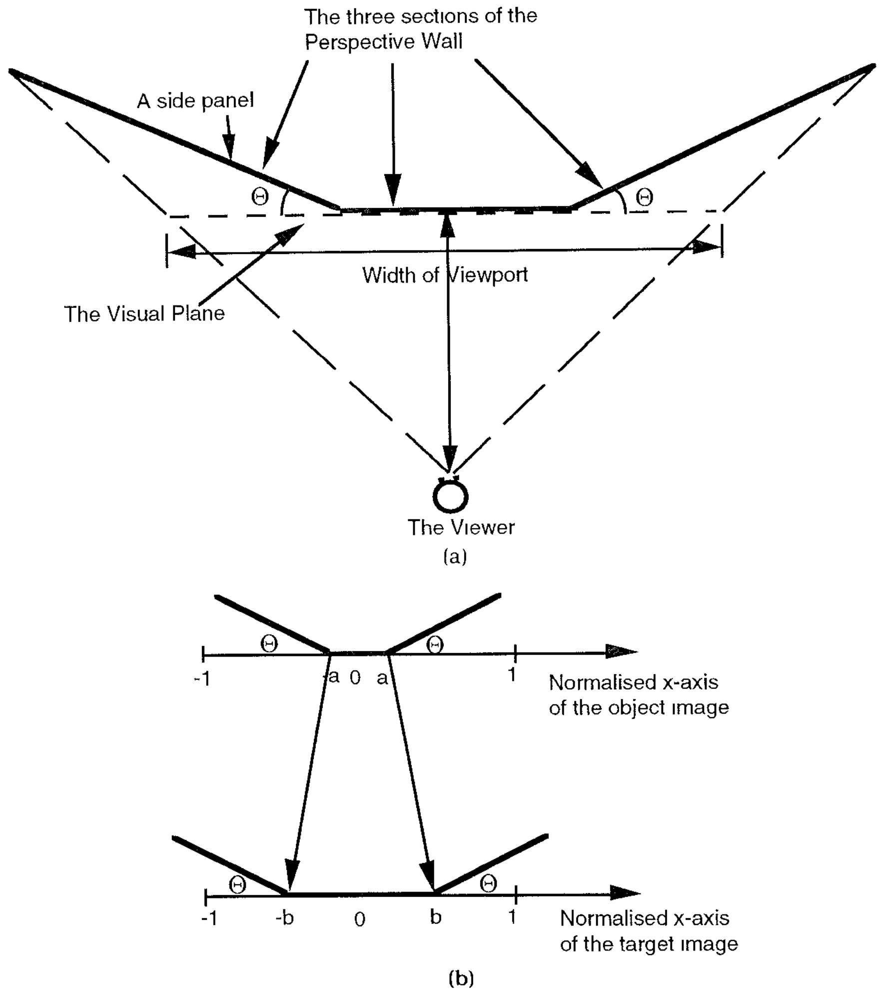

Fig. A.2. (a) The physical arrangement of the Perspective Wall; (b) the relationships between the object image and target image.

of the Perspective Wall, and θ, the angle between the side panel of the Perspective Wall and the visual plane (Figure A.2).

The position of V can be determined by equating the gradient of the two line segments: $\mathbf{V} - \mathbf{(1,0)}$ and $\mathbf{(1,0)} - \mathbf{E}$ (see Figure A.3):

$$\frac{-V_y - 0}{0 - 1} = \frac{0 - (1 - a).\sin\theta}{1 - [b + (1 - a).\cos\theta]}$$

$$V_y = \frac{-(1 - a).\sin\theta}{1 - [b + (1 - a).\cos\theta]}. \tag{7}$$

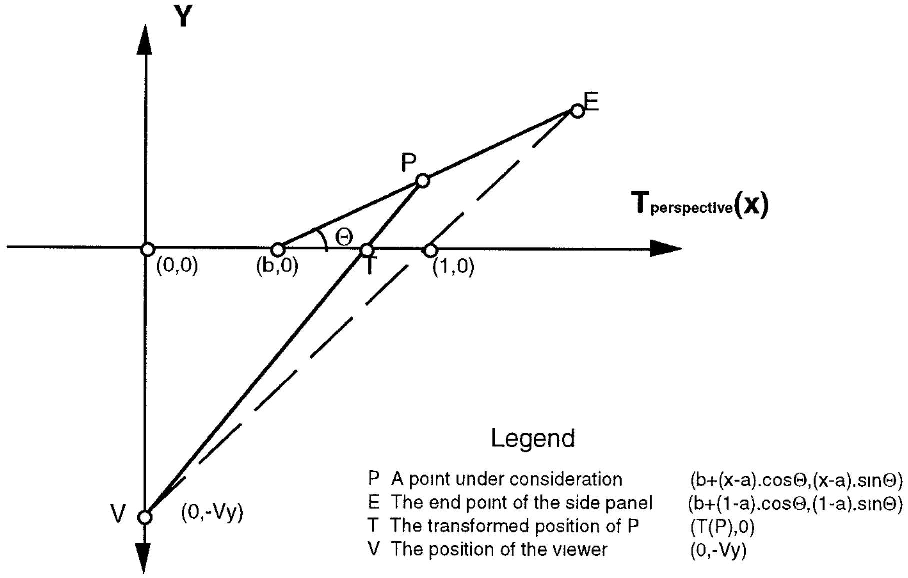

Fig. A.3. A simplified elevated view of the Perspective Wall.

Now, the transformed position of a point **P**, $\mathbf{T_X(P)}$, can be determined by equating the gradient of the two line segments: $\mathbf{T_X(P) - V}$ **and** $\mathbf{P - V}$,

$$\frac{T_x - 0}{0 - (-V_y)} = \frac{b + (x - a).\cos\theta - 0}{(x - a).\sin\theta - (-V_y)}$$

$$T_x = \frac{V_y[b + (x - a).\cos\theta]}{V_y + (x - a).\sin\theta}. \tag{8}$$

Substituting (7) in (8), we have

$$T_x = \frac{\dfrac{-(1 - a).\sin\theta}{1 - [b + (1 - a).\cos\theta]}.[b + (x - a).\cos\theta]}{\dfrac{-(1 - a).\sin\theta}{1 - [b + (1 - a).\cos\theta]} + (x - a).\sin\theta}$$

$$T_x = \frac{-(1 - a).\sin\theta.[b + (x - a).\cos\theta]}{-(1 - a).\sin\theta + (x - a).\sin\theta.\{1 - [b + (1 - a).\cos\theta]\}}. \tag{9}$$

Dividing the numerator and denominator of (9) by $\sin\theta$, we have

$$T_x = \frac{-(1 - a).[b + (x - a).\cos\theta]}{-(1 - a) + (x - a).\{1 - [b + (1 - a).\cos\theta]\}}. \tag{10}$$

Dividing the numerator and denominator of (10) by $-(1 - a)$ and regrouping, we have

$$T_x = \frac{-(1-a).[b + (x-a).\cos\theta]}{-(1-a) + [(1-b) - (1-a).\cos\theta].(x\text{-}a)}$$

$$= \frac{[b + (x-a).\cos\theta]}{1 - \left[\dfrac{(1-b)}{(1-a)} - \cos\theta\right].(x-a)}.$$

The transformation functions for a general Perspective Wall are therefore

$$\text{for } x \le a. \quad T_{perspective}(x) = x.\frac{b}{a}, \tag{11}$$

$$\text{for } x > a, \quad T_{perspective}(x) = T_x = \frac{[b + (x-a).\cos\theta]}{1 - \left[\dfrac{(1-b)}{(1-a)}\cos\theta\right].(x-a)}. \tag{12}$$

It should be noted that in Mackinlay et al.'s implementation of the Perspective Wall $a = b$, and hence (11) and (12) become

$$\text{for } x \le a, \quad T_{perspective}(x) = x,$$

$$\text{for } x > a, \quad T_{perspective}(x) = \frac{[a + (x-a).\cos\theta]}{1 - [1 - \cos\theta].(x-a)}.$$

By definition the magnification function of a general Perspective Wall is given by

$$M_{perspective}(x) = \frac{d}{dx}\left(T_{perspective}(x)\right),$$

$$\text{for } x \le a, \quad M_{perspective}(x) = \frac{b}{a} \tag{13}$$

$$\text{for } x > a, \quad M_{perspective}(x) = \frac{d}{dx}\left(\frac{[b + (x-a).\cos\theta]}{1 - \left[\dfrac{(1-b)}{(1-a)} - \cos\theta\right].(x-a)}\right). \tag{14}$$

Let $k = (1 - b)/(1 - a)$, and simplifying (14), we have

$$M_{perspective}(x) = \frac{d}{dx}\left(\frac{[b + (x-a).\cos\theta]}{1 - [k - \cos\theta].(x-a)}\right)$$

$$= \frac{\cos\theta}{[1 - (k - \cos\theta).(x-a)]}$$

$$+ \frac{[b + (x-a).\cos\theta].[k - \cos\theta]}{[1 - (k - \cos\theta).(x-a)]^2}.$$

Table A.II. A Summary of Transformation and Magnification Functions

	Transformation Function T(x)	Magnification Function M(x)
Polyfocal Projection	$x+\frac{A.x}{(1+C.x^2)}$	$1+\frac{A.(1-C.x^2)}{(1+C.x^2)^2}$
Fisheye View	$\frac{(1+d).x}{(d.x+1)}$	$\frac{d+1}{(d.x+1)^2}$
Perspective Wall		
for $x \le a$,	$x.\frac{b}{a}$	$\frac{b}{a}$
for $x > a$,	$\frac{[b+(x-a).\cos\theta]}{1-[\frac{(1-b)}{(1-a)}-\cos\theta].(x-a)}$	$\frac{b.k+(1-b).\cos\theta}{[(k-\cos\theta).x+(a.\cos\theta-a.k-1)]^2}$ note: $k=\frac{(1-b)}{(1-a)}$
Bifocal Display		
for $x \le a$,	$x.\frac{b}{a}$	$\frac{b}{a}$
for $x > a$,	$b+(x-a).\frac{(1-b)}{(1-a)}$	$\frac{(1-b)}{(1-a)}$

Simplifying and regrouping, we have

$$M_{perspective}(x) = \frac{b.k + (1-b).\cos\theta}{[1-(k-\cos\theta).(x-a)]^2}$$
$$= \frac{b.k + (1-b).\cos\theta}{[1-(k-\cos\theta).x + (k-\cos\theta).a]^2}$$
$$= \frac{b.k + (1-b).\cos\theta}{[(k-\cos\theta).x + (a.\cos\theta - a.k - 1)]^2}. \tag{15}$$

With $a = b$, and therefore $k = 1$, the magnification functions of Mackinlay et al.'s implementation of the Perspective Wall become

$$\text{for } x \leq a, \quad M_{perspective}(x) = 1$$

$$\text{for } x > a, \quad M_{perspective}(x) = \frac{a + (1 - a).\cos\theta}{[(1 - \cos\theta).x + (a.\cos\theta - a - 1)]^2}.$$

A4. Fisheye View [Sarkar and Brown 1992]

The transformation function of the Fisheye View is given by

$$T_{fisheye}(x) = \frac{1 + d}{\left(d + \dfrac{1}{x}\right)}$$

$$= \frac{(1 + d).x}{(d.x + 1)} \tag{16}$$

where d is called the distortion factor. The magnification function of the Fisheye View is, therefore,

$$M_{fisheye}(x) = \frac{d}{dx}\left(T_{fisheye}(x)\right)$$

$$= \frac{(1 + d).(d.x + 1) - (1 + d).x.d}{(d.x + 1)^2}$$

$$= \frac{(d.x + 1) + d^2.x + d - x.d - x.d^2}{(d.x + 1)^2}$$

$$= \frac{d + 1}{(d.x + 1)^2}. \tag{17}$$

A5. Summary

Table A.II summarizes the transformation and magnification functions of the distortion-oriented techniques derived earlier.

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REFERENCES

APPERLEY, M. D., TZAVARAS, I., AND SPENCE, R. 1982. A bifocal display technique for data presentation. In *Proceedings of Eurographics '82*. 27–43.

BOLT, R. A. 1979. *Spatial Data Management.* Architecture Machine Group, MIT, Cambridge, Mass.

BURGER, S. V., AND APPERLEY, M. D. 1991. A multi-dimensional approach to interface evaluation. In *Proceedings of the IFIP Conference on Human Jobs and Computer Interface-WG9.1.* IFIP, 205–222.

BEARD, D. V., AND WALKER J. Q., II. 1990. Navigational techniques to improve display of large two-dimensional spaces. *Behav. Inf. Tech. 9*, 451–466.

CARD, S. K., ROBERTSON, G. G., AND MACKINLAY, J. D. 1991. The information visualizer, an information workspace. In *Proceedings of CHI '91.* ACM, New York, 181–188.

CHEN, H. C., AND CHAN, K. T. 1990. Reading computer-displayed moving text with and without self-control over the display rate. *Behav. Inf. Tech. 9*, 467–477.

DONELSON, W. 1978. Spatial management of information. In *ACM SIGGRAPH '78 Proceedings.* ACM, New York, 203–209.

FARRAND, W. A. 1973. Information display in interactive design. Ph.D. dissertation, Dept. of Engineering, Univ. of California, Los Angeles, Calif.

FURNAS, G. W. 1986. Generalized fisheye views. In *Proceedings of CHI '86.* ACM, New York, 16–23.

HEROT, C. F., CARLING, R., FRIEDELL, M., AND FRAMLICH, D. 1980. A prototype spatial data management system. *Comput. Graph. 14*, 1, 63–70.

HOLLANDS, J. G., CAREY, T. T., MATTHEWS, M. L., AND MCCANN, C. A. 1989. Presenting a graphical network: A comparison of performance using fisheye and scrolling views. In *Designing and Using Human-Computer Interfaces and Knowledge Based Systems*, G. Salvendy and M. Smith, Eds. Elsevier, Amsterdam, 313–320.

JOHNSON, B., AND SHNEIDERMAN, B. 1991. Tree maps: A space-filling approach to the visualisation of hierarchical information structures. In *Proceedings of the 2nd International IEEE Visualisation Conference.* IEEE, New York, 284–291.

KADMON, N., AND SHLOMI, E. 1978. A polyfocal projection for statistical surfaces. *Cartograph. J. 15*, 1, 36–41.

KANG, T. J., AND MUTER, P. 1989. Reading dynamic displayed text. *Behav. Inf. Tech. 8*, 1, 33–42.

KOIKE, H. 1994. Fractal views: A fractal-based method for controlling information display. *ACM Trans. Inf. Syst.* To be published.

LEUNG, Y. K. 1989. Human-computer interaction techniques for map-based diagrams. In *Designing and Using Human-Computer Interfaces and Knowledge Based Systems*, G. Salvendy and M. Smith, Eds. Elsevier, Amsterdam, 361–368.

LEUNG, Y. K., AND APPERLEY, M. D. 1993a. Extending the Perspective Wall. In *Proceedings of OZCHI '93.* 110–120.

LEUNG, Y. K., AND APPERLEY, M. D. 1993b. E^3: Towards the metrication of graphical presentation techniques. In *Lecture Notes in Computer Science: Human-Computer Interaction*, Bass, L. J., Gornostaev, J. G., and Unger, C., Eds. Springer-Verlag, Heidelberg, 125–140.

MACKINLAY, J. D., ROBERTSON, G. G., AND CARD, S. K. 1991. The Perspective Wall: Detail and context smoothly integrated, In *Proceedings of CHI '91.* ACM, New York, 173–179.

MISUE, K., AND SUGIYAMA, K. 1991. Multi-viewpoint perspective display methods: Formulation and application to compound graphs. In *Human Aspects in Computing: Design and Use of Interactive Systems and Information Management*, H. J. Bullinger, Ed. Elsevier Science Publishers B. V., Amsterdam, 834–838.

MITTA, D. A. 1990. A fisheye presentation strategy: Aircraft maintenance data. In *Proceedings of Interact '90.* 875–885.

MONK, F. M., WALSH, P., AND DIX, A. J. 1988. A comparison of hypertext, scrolling and folding as mechanisms for program browsing. In *People and Computers.* Vol. 4, D. M. Jones and R. Winder, Eds. Cambridge University Press, Cambridge, Mass., 421–435.

RAO, R., AND CARD, S. K. 1994. The Table Lens: Merging graphical and symbolic representations in an interactive focus + context visualization for tabular information. In *Proceedings of CHI'94.* ACM, New York.

ROBERTSON, G. G., AND MACKINLAY, J. D. 1993. The Document Lens. In *Proceedings of the ACM Symposium on User Interface Software and Technology.* ACM, New York.

ROBERTSON, G. G., MACKINLAY, J. D., AND CARD, S. K. 1991. Cone Trees: Animated 3D visualizations of hierarchical information. In *Proceedings of CHI '91.* ACM, New York, 189–194.

SARKAR, M., AND BROWN, M. H. 1992. Graphical fisheye views of graphs. In *Proceedings of CHI'92*. ACM, New York, 83–91.

SARKAR, M., SNIBBE, S., AND REISS, S. 1993. Stretching rubber sheet: A metaphor for visualising large structure on small screen. In *Proceedings of the ACM Symposium on User Interface Software and Technology*. ACM, New York.

SCHAFFER, D., ZUO, Z., BARTRAM, L., DILL, J., DUBS, S., GREENBERG, S., AND ROSEMAN, M. 1993. Comparing fisheye and full-zoom techniques for navigation of hierarchically clustered networks. In *Proceedings of Graphics Interface '93*. Morgan-Kaufman, San Mateo, Calif.

SHNEIDERMAN, B. 1992. Tree visualization with Tree-Maps: 2-d space-filling approach. *ACM Trans. Comput. Graph. 11*, 1, 92–97.

SPENCE, R., AND APPERLEY, M. D. 1982. Database navigation: An office environment for the professional. *Behav. Inf. Tech. 1*, 1, 43–54.

TOBLER, W. R. 1973. A continuous transformation useful for districting. *Ann. New York Acad. Sci. 219*, 215–220.