Smart Graphics: Methoden 3
Suche, Constraints

Vorlesung „Smart Graphics”
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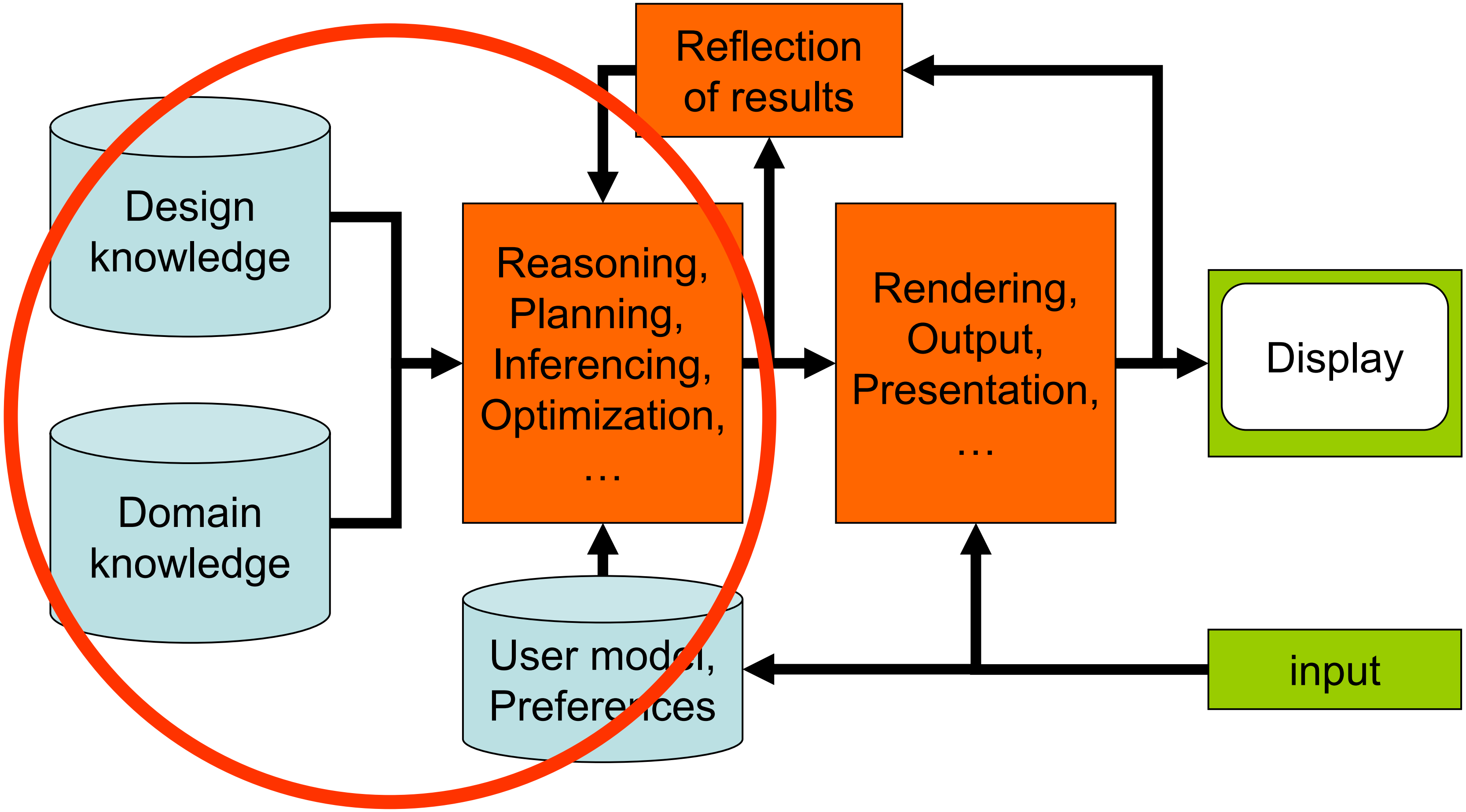
Mittwoch, 30. November 2005

Themen heute

- Suchverfahren
  - Alpha-Beta
  - Hillclimbing
  - Simulated Annealing
  - Genetische Suche
- Constraints



Reflection of results

Design knowledge

Domain knowledge

Reasoning, Planning, Inferencing, Optimization, …

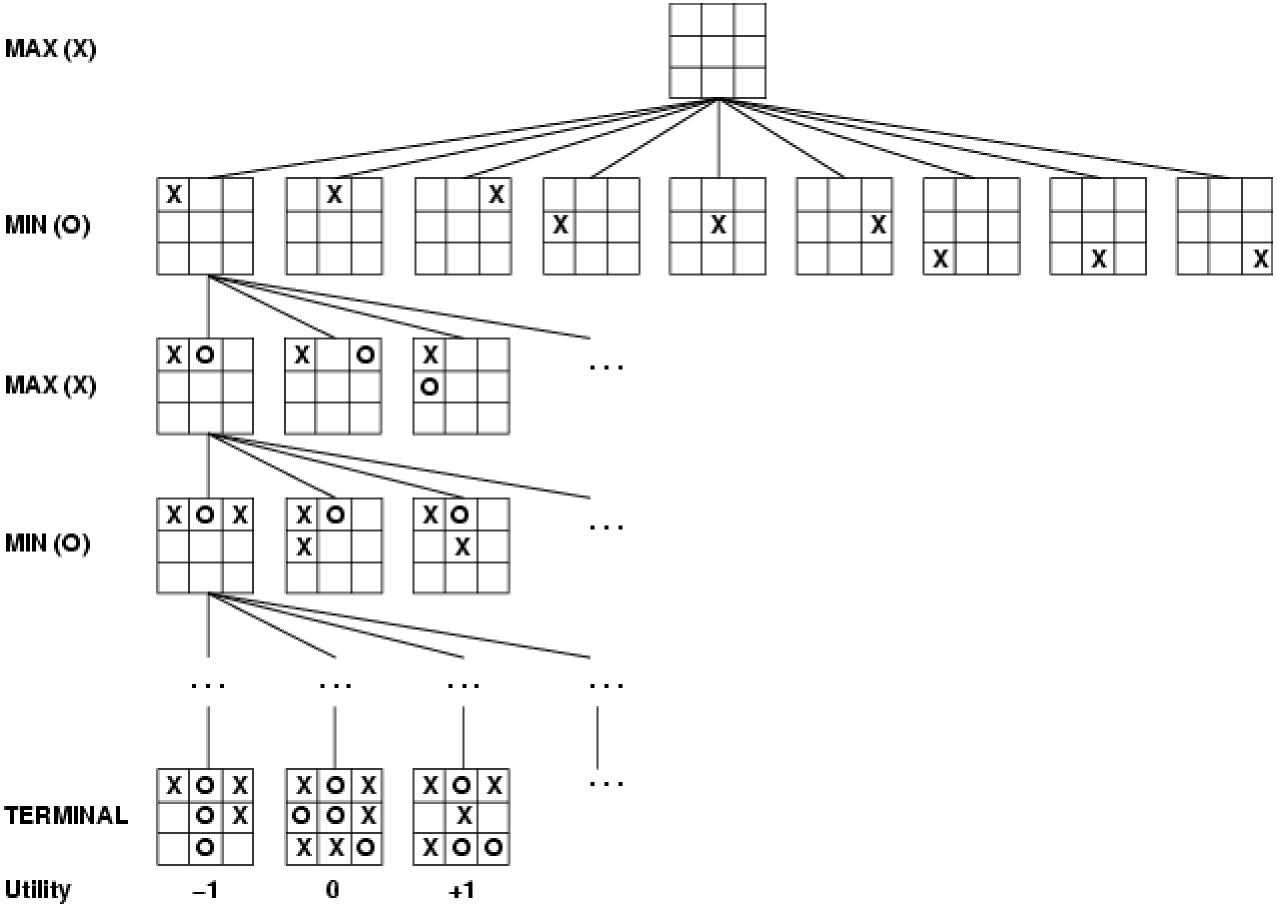
Rendering, Output, Presentation, …

Display

User model, Preferences

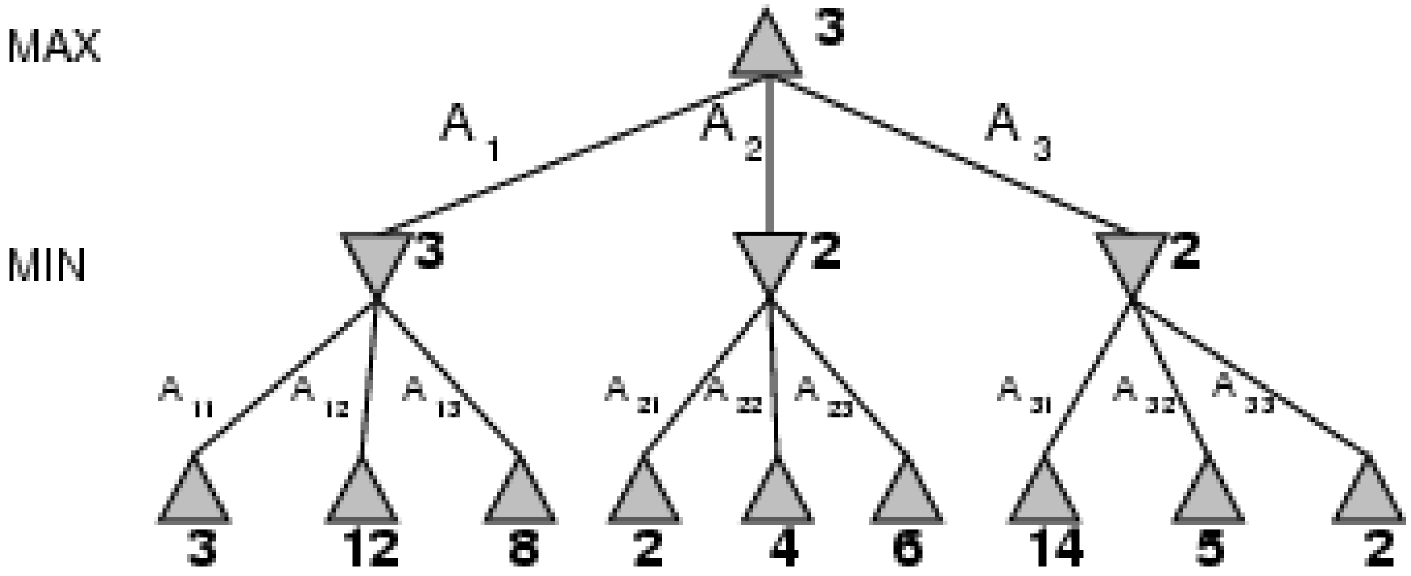
input

Game tree (2-player, deterministic, turns)

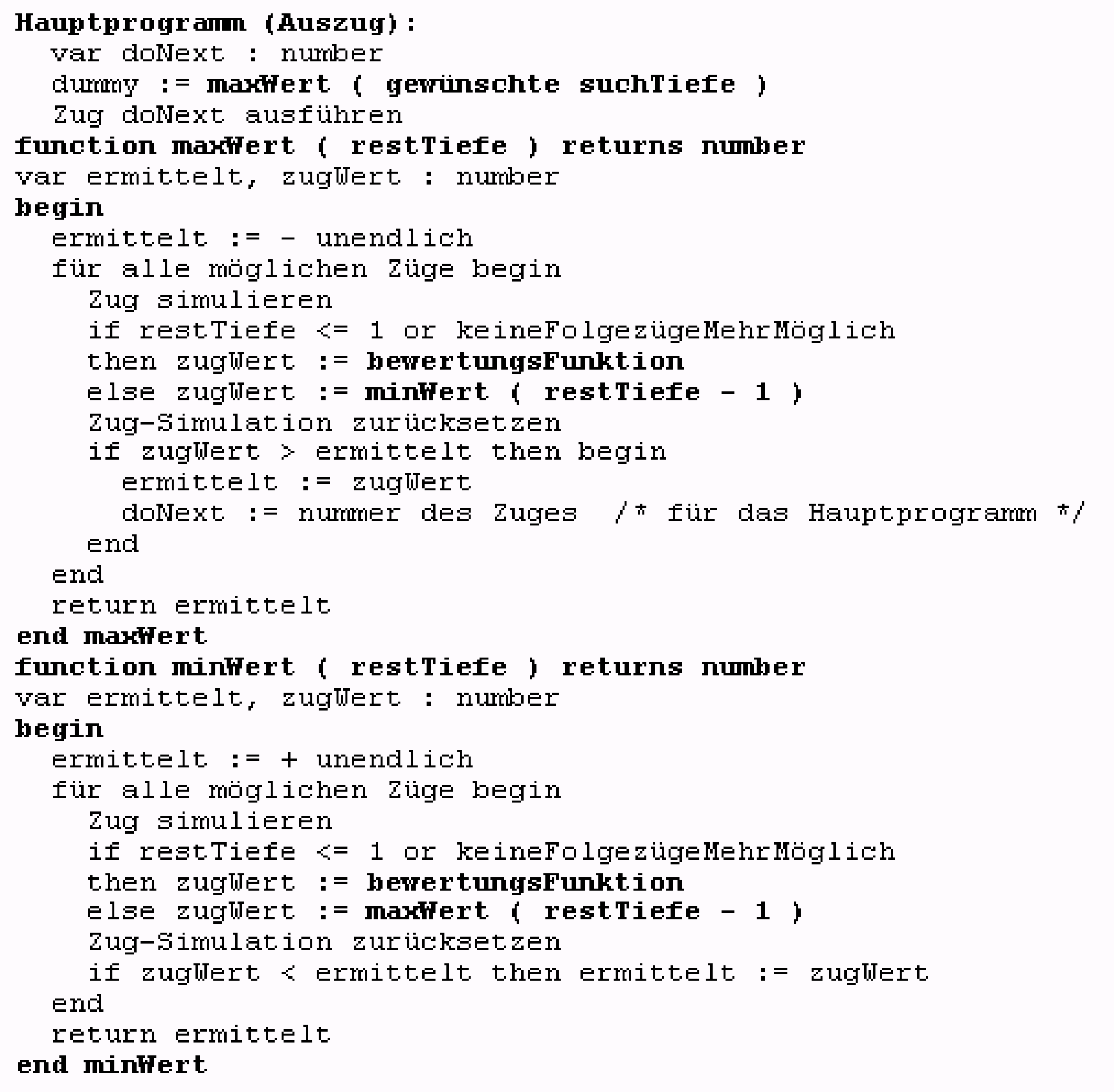


Minimax

- Perfect play for deterministic games.
- Idea: choose move to position with highest minimax value = best achievable payoff against best play.
- E.g., 2-ply game:



Minimax algorithm



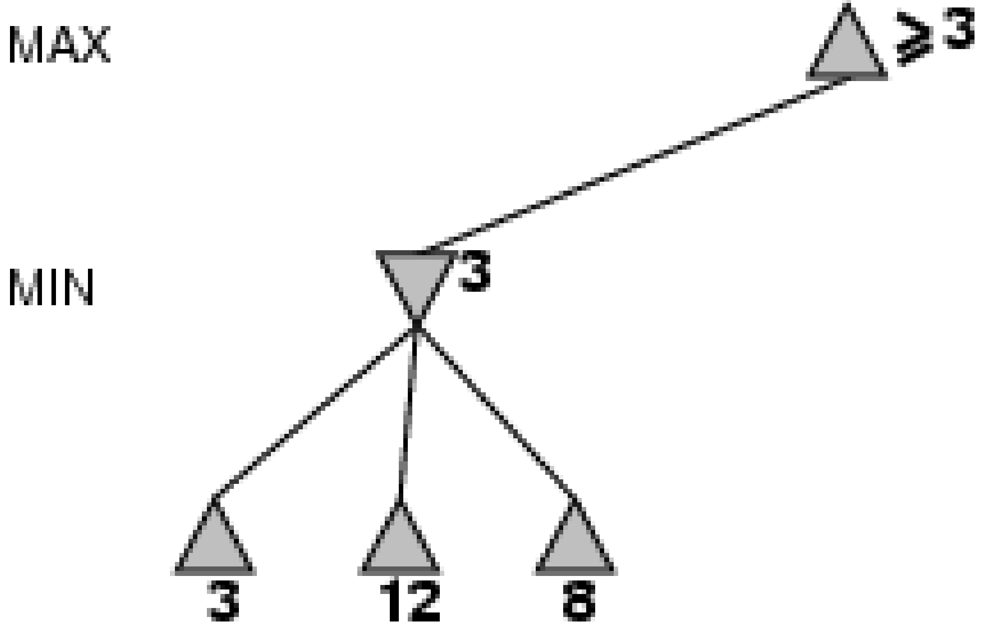
```
Hauptprogramm (Auszug):
  var doNext : number
  dummy := maxWert ( gewünschte suchTiefe )
  Zug doNext ausführen
function maxWert ( restTiefe ) returns number
var ermittelt, zugWert : number
begin
  ermittelt := - unendlich
  für alle möglichen Züge begin
    Zug simulieren
    if restTiefe <= 1 or keineFolgezügeMehrMöglich
    then zugWert := bewertungsFunktion
    else zugWert := minWert ( restTiefe - 1 )
    Zug-Simulation zurücksetzen
    if zugWert > ermittelt then begin
      ermittelt := zugWert
      doNext := nummer des Zuges  /* für das Hauptprogramm */
    end
  end
  return ermittelt
end maxWert
function minWert ( restTiefe ) returns number
var ermittelt, zugWert : number
begin
  ermittelt := + unendlich
  für alle möglichen Züge begin
    Zug simulieren
    if restTiefe <= 1 or keineFolgezügeMehrMöglich
    then zugWert := bewertungsFunktion
    else zugWert := maxWert ( restTiefe - 1 )
    Zug-Simulation zurücksetzen
    if zugWert < ermittelt then ermittelt := zugWert
  end
  return ermittelt
end minWert
```

Properties of minimax

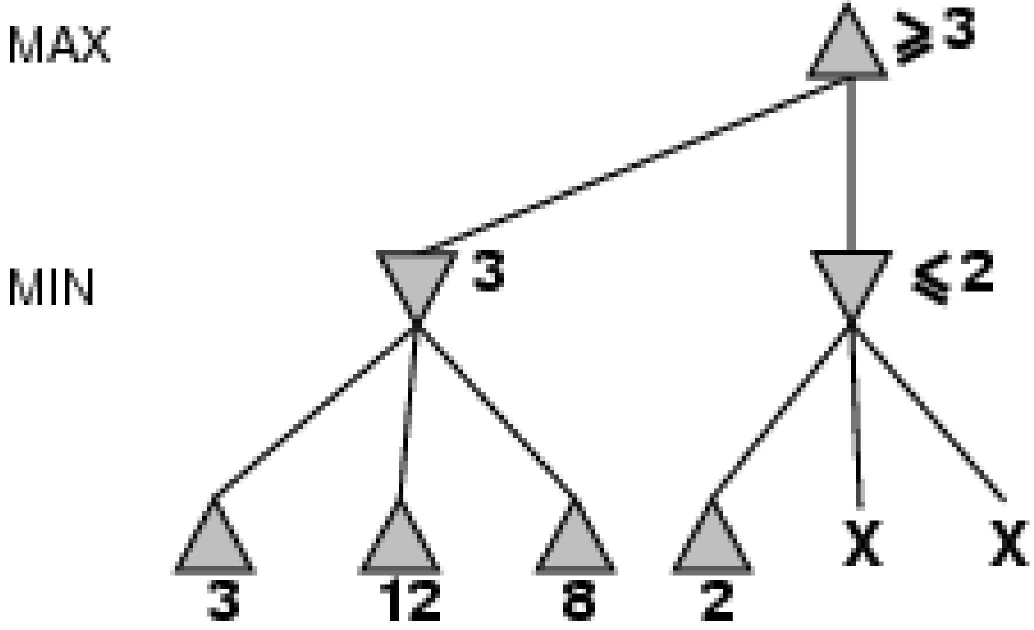
- Complete? Yes (if tree is finite).
- Optimal? Yes (against an optimal opponent).
- Time complexity? O(b^m).
- Space complexity? O(bm) (depth-first exploration).

- For chess, b ≈ 35, m ≈100 for "reasonable" games
→ exact solution completely infeasible.

α-β pruning example



α-β pruning example



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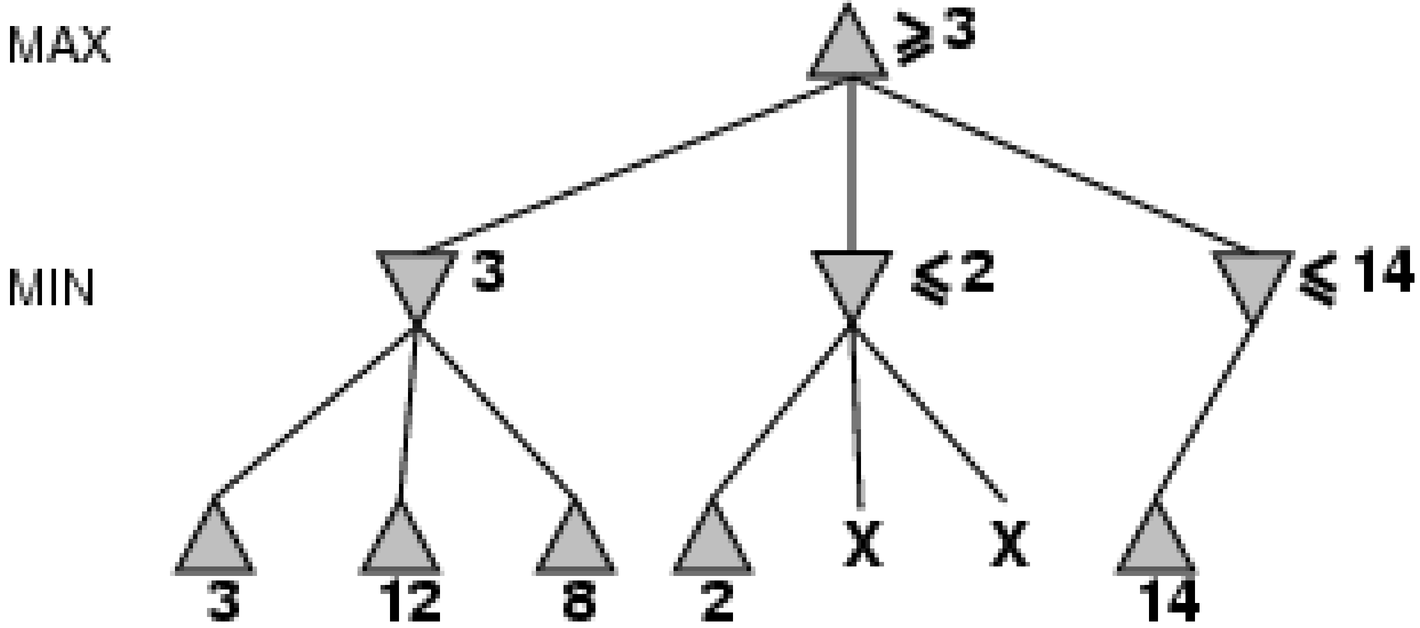

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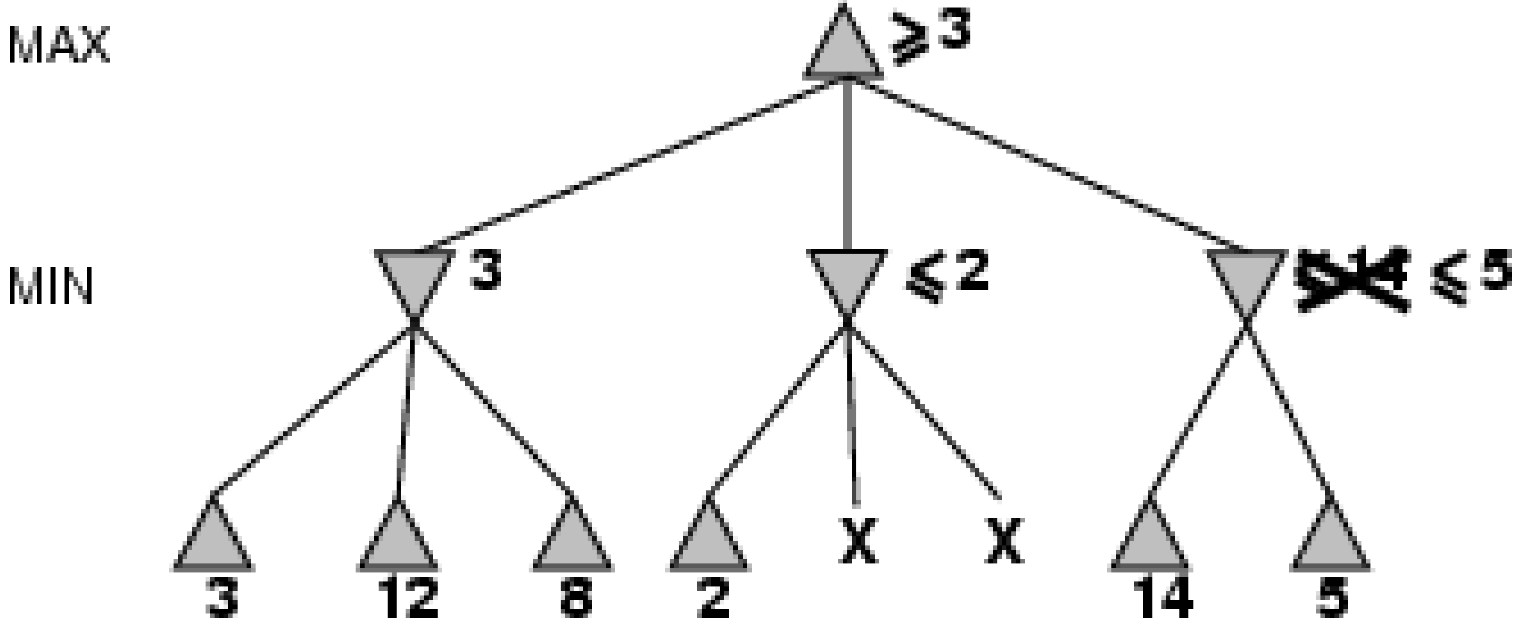

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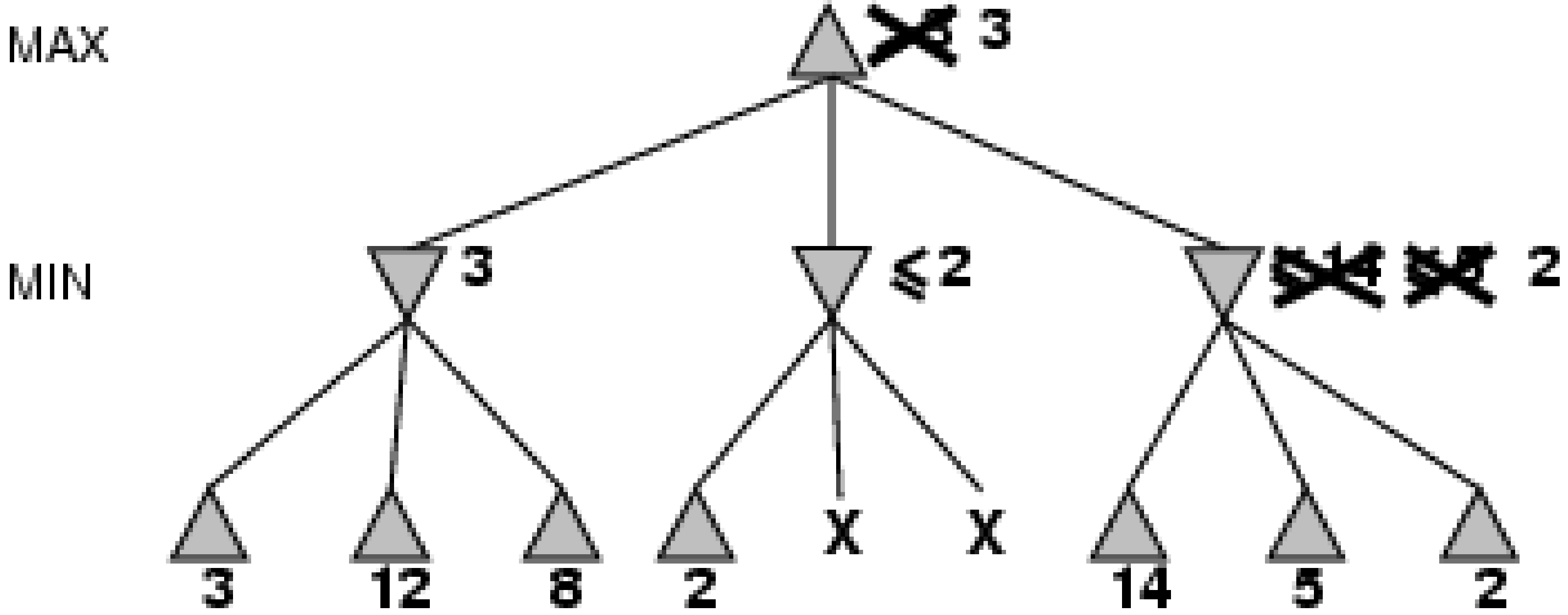


Properties of α-β

- Pruning does not affect final result.

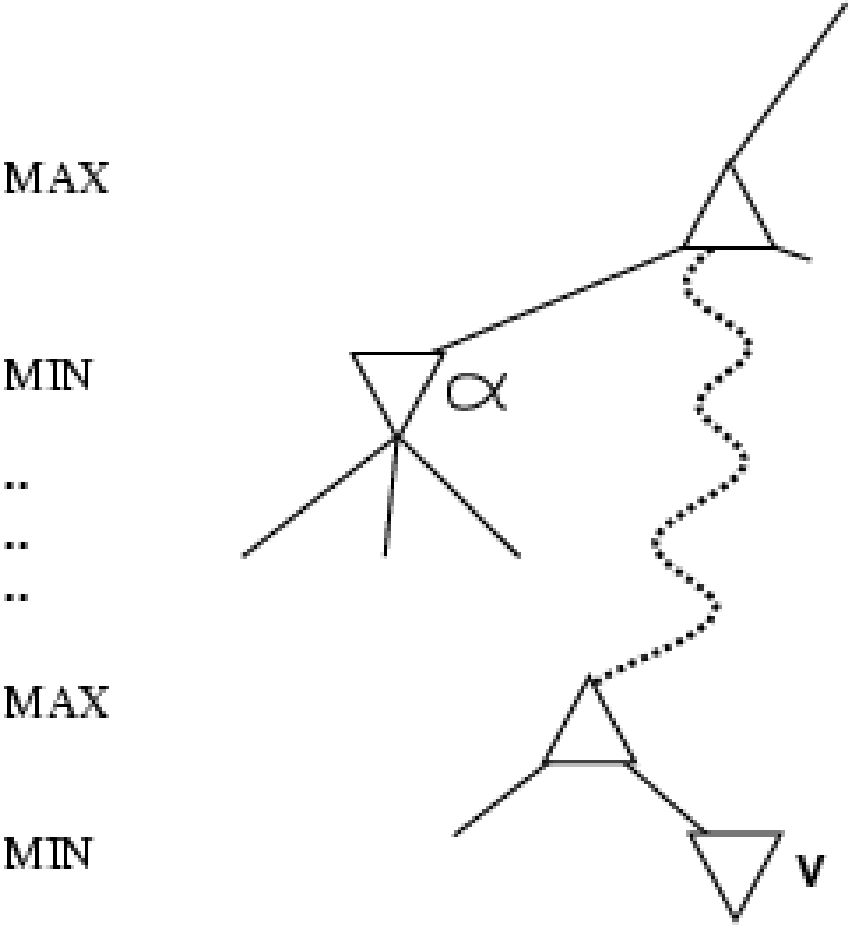
- Good move ordering improves effectiveness of pruning.

- With "perfect ordering," time complexity = O($b^{m/2}$)
  → doubles depth of search.

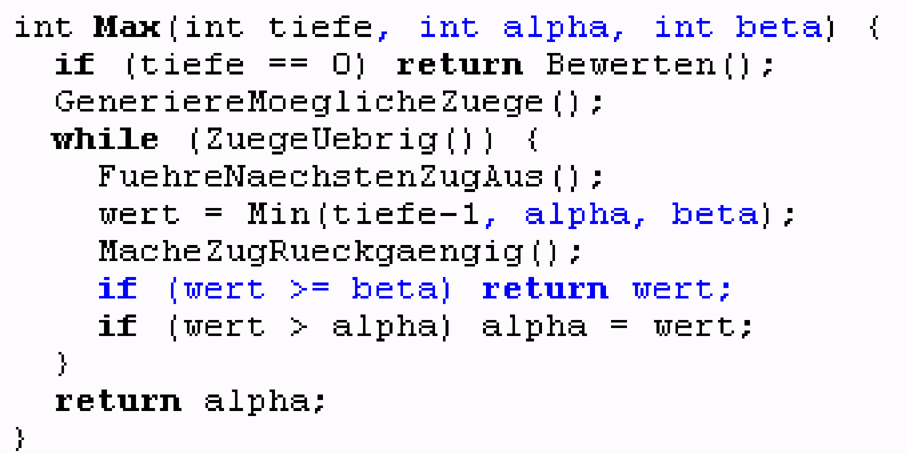
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning).

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*.
- If *v* is worse than α, *max* will avoid it.
  - → prune that branch.
- Define β similarly for *min*.

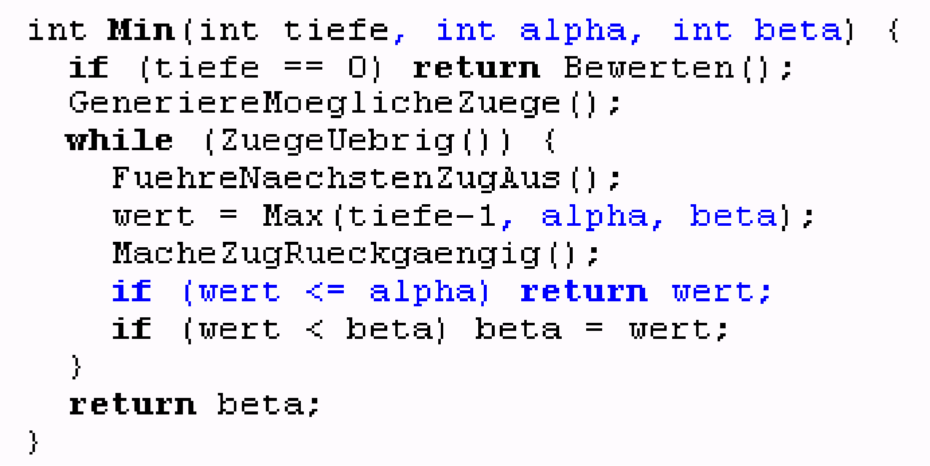


The α-β algorithm



```
int Max(int tiefe, int alpha, int beta) {
  if (tiefe == 0) return Bewerten();
  GeneriereMoeglicheZuege();
  while (ZuegeUebrig()) {
    FuehreNaechstenZugAus();
    wert = Min(tiefe-1, alpha, beta);
    MacheZugRueckgaengig();
    if (wert >= beta) return wert;
    if (wert > alpha) alpha = wert;
  }
  return alpha;
}
```

The α-β algorithm



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  while (ZuegeUebrig()) {
    FuehreNaechstenZugAus();
    wert = Max(tiefe-1, alpha, beta);
    MacheZugRueckgaengig();
    if (wert <= alpha) return wert;
    if (wert < beta) beta = wert;
  }
  return beta;
}
```

Zusammenfassung

Ausgangspunkt: Bei der Tiefensuche schon *während der Expansion die Bewertungsfunktion auf Blätter anwenden* und Werte nach Minimax-Verfahren nach oben verteilen.

- *Alpha-Wert eines Max-Knotens* ist die jeweils größte geerbte Bewertung seiner Nachfolger. Der Wert den Spieler Max mindestens erhält.
- *Beta-Wert eines Min-Knotens* ist der jeweils kleinste geerbte Wert seiner Nachfolger. Der Wert den Spieler Min maximal erhält.

- 2 Typen von Beschneidungen des Suchbaums:
  - *Alpha-Schnitt*: Suche wird abgebrochen an einem *Min-Knoten*, dessen *Beta-Wert > Alpha-Wert* von irgendeinem seiner *Max-Vorgänger*. Der Beta-Wert bleibt der vererbte Wert des Min-Knotens, der gemäß Minimax weiterverarbeitet wird.
  - *Beta-Schnitt*: Suche wird abgebrochen an einem *Max-Knoten*, dessen *Alpha-Wert < Beta-Wert* von irgendeinem seiner *Min-Vorgänger*. Der Alpha-Wert bleibt der vererbte Wert des Max-Knotens.

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal* is replaced by *Cutoff*
2. *Utility* is replaced by *Eval*.

Does it work in practice?.

$b^m = 10^6$, b=35 → m=4.

4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice.
- 8-ply ≈ typical PC, human master.
- 12-ply ≈ Deep Blue, Kasparov.

Deterministic games in practice

- Checkers:
  - Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a pre-computed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

Deterministic games in practice

- Chess:
  - Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.
  - Deep Fritz achieved remis twice against Kasparov and Kramnik in 2002/03
  - Hydra defeated Adams (ranked No. 7) in 2005 with 1:5
  - Strategy vs. Nr. of computed positions.

- Go:
  - human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Local search algorithms

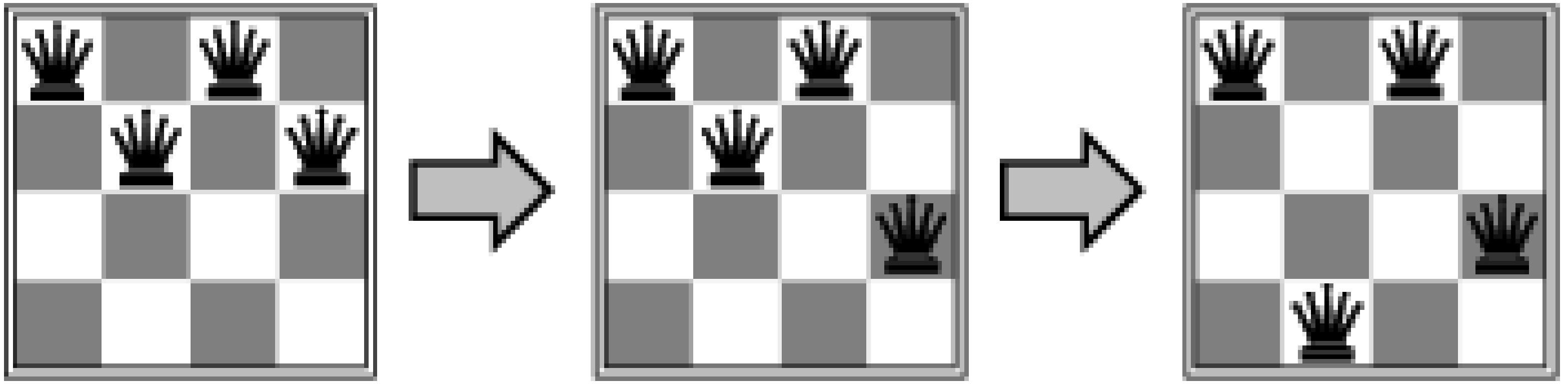
- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution.

- State space = set of "complete" configurations.
- Find configuration satisfying constraints, e.g., n-queens.

- In such cases, we can use local search algorithms.
- keep a single "current" state, try to improve it.

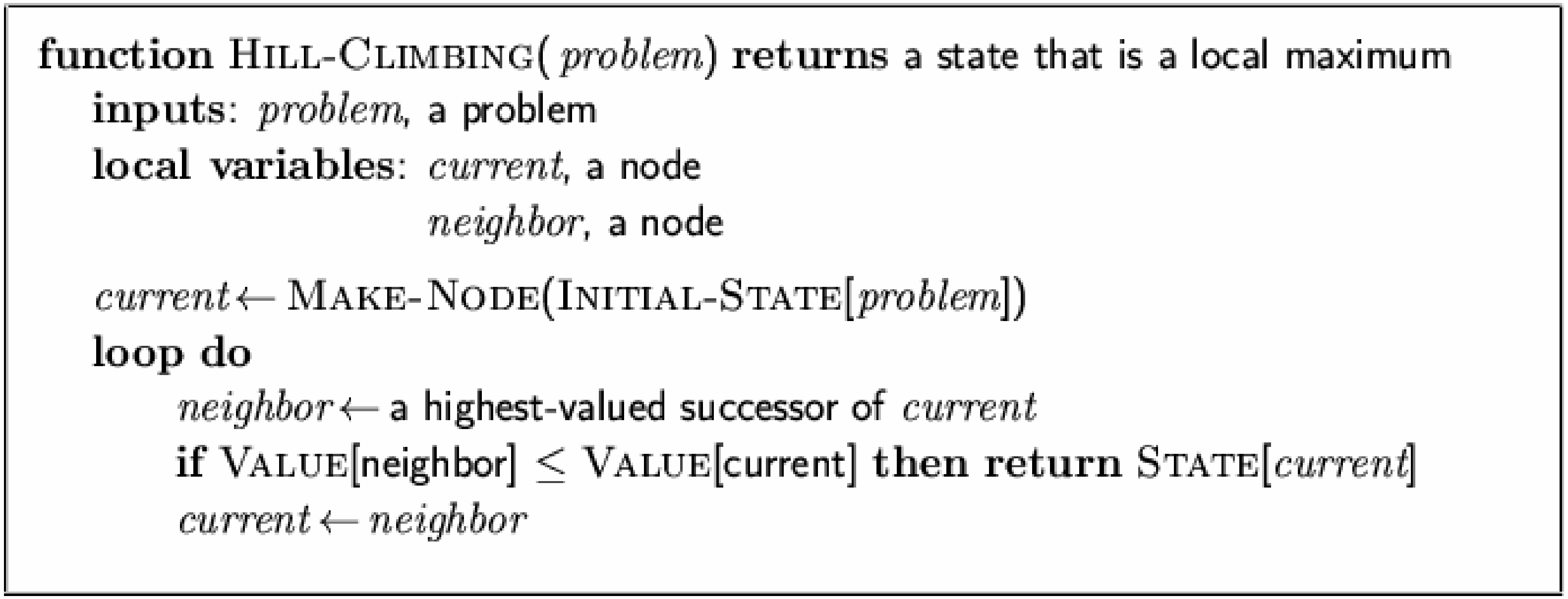
Example: *n*-queens

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal



Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

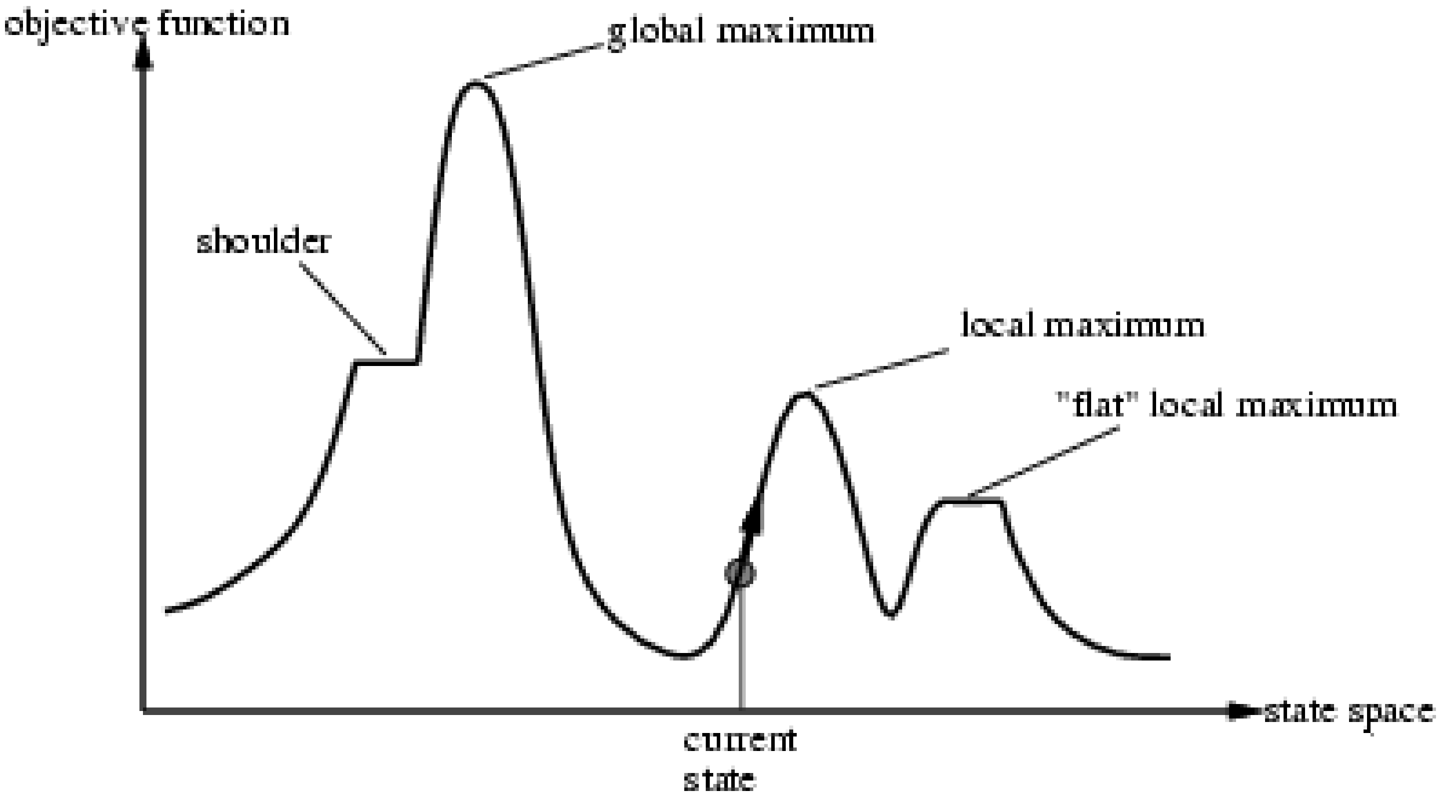


```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

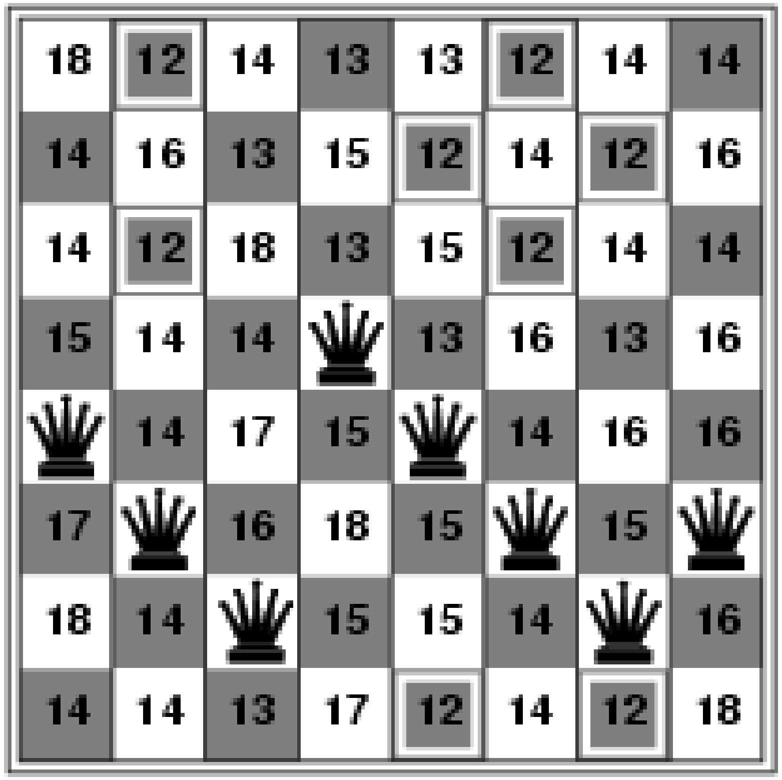
    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima



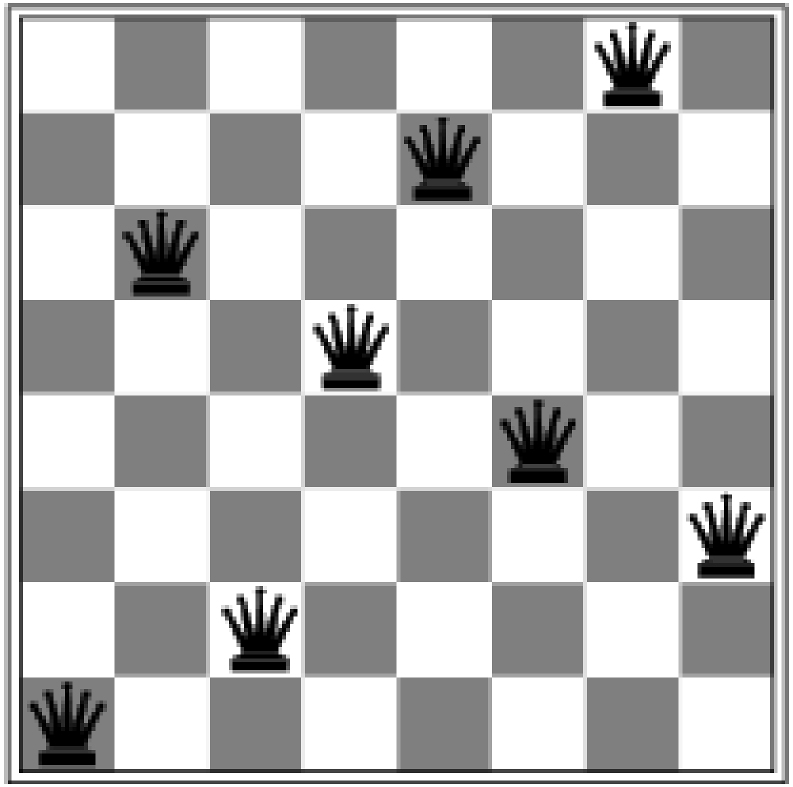
Hill-climbing search: 8-queens problem



18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♛	13	16	13	16
♛	14	17	15	♛	14	16	16
17	♛	16	18	15	♛	15	♛
18	14	♛	15	15	14	♛	16
14	14	13	17	12	14	12	18

- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state

Hill-climbing search: 8-queens problem



- A local minimum with *h = 1*

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- The algorithm employs a random search which not only accepts changes that decrease objective function **f**, but also some changes that increase it. The latter are accepted with a probability $p = exp\left(-\frac{\delta f}{T}\right)$

Properties of simulated annealing search

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

- Widely used in VLSI layout, airline scheduling, etc.
- Adaptation of values for T is application driven.

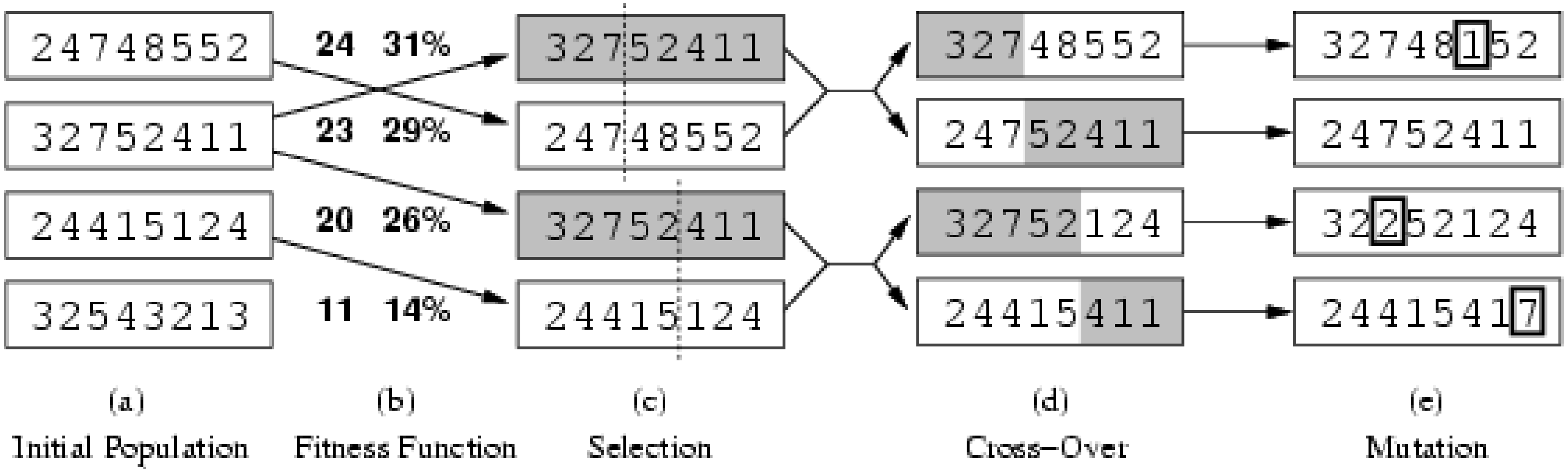
Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Genetic algorithms

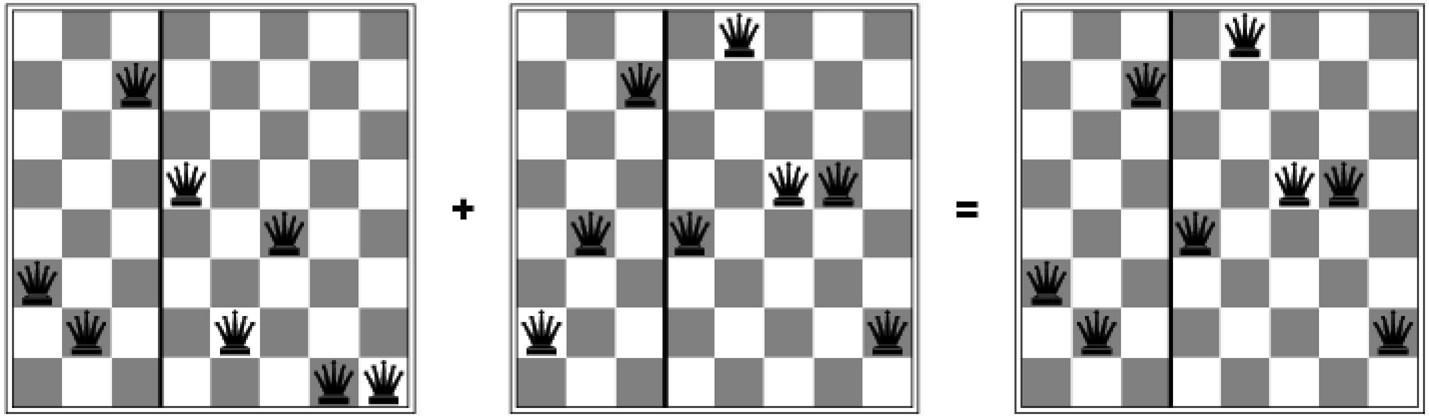
- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms



Constraint Satisfaction Problems

- Constraint Satisfaction Problems (CSP)
- Backtracking search for CSPs
- Local search for CSPs

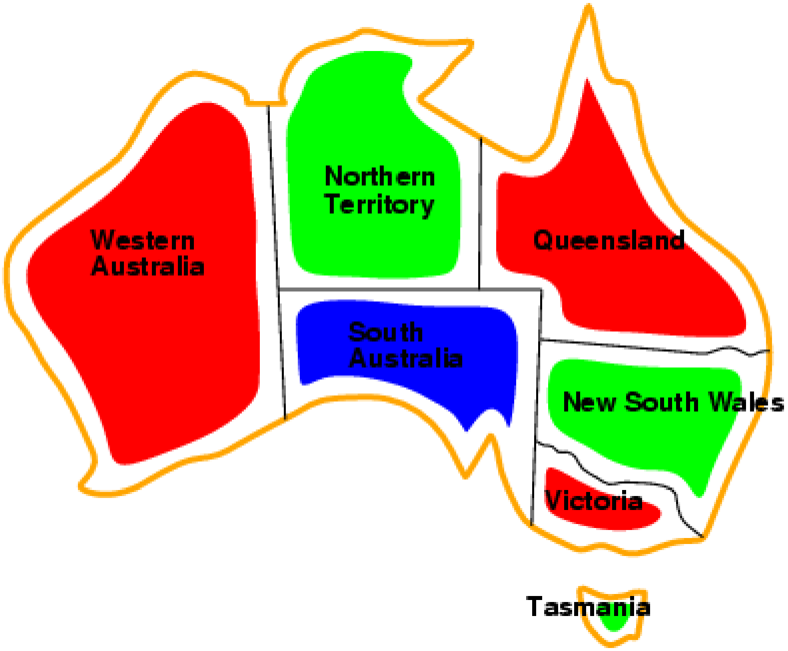
Constraint satisfaction problems (CSPs)

- Standard search problem:
  - state is a "black box“ – any data structure that supports successor function, heuristic function, and goal test
- CSP:
  - state is defined by variables X_i with values from domain D_i
  - goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a formal representation language
- Allows useful general-purpose algorithms with more power than standard search algorithms

Example: Map-Coloring

- Variables *WA, NT, Q, NSW, V, SA, T*
- Domains D_i = {red,green,blue}
- Constraints: adjacent regions must have different colors
- e.g., WA ≠ NT, or (WA,NT) in {(red,green),(red,blue),(green,red), (green,blue),(blue,red),(blue,green)}

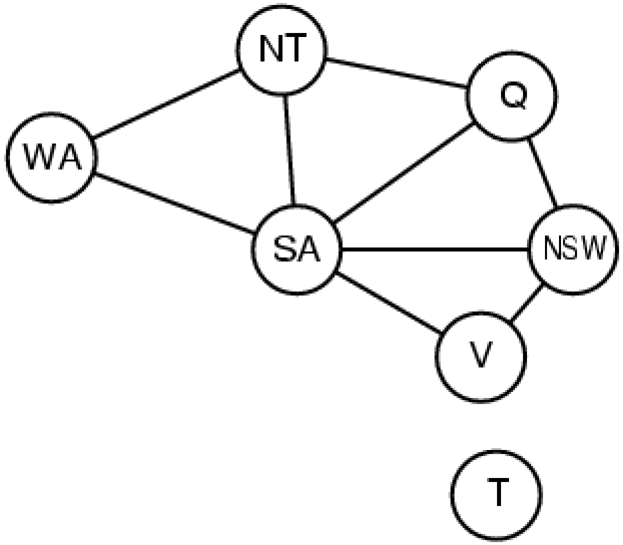
Example: Map-Coloring



- Solutions are complete and consistent assignments, e.g., WA = red, NT = green,Q = red,NSW = green,V = red,SA = blue,T = green

Constraint graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints



Varieties of CSPs

- Discrete variables
  - finite domains:
    - n variables, domain size d → $O(d^n)$ complete assignments
    - e.g., Boolean CSPs, incl.~Boolean satisfiability (NP-complete)
  - infinite domains:
    - integers, strings, etc.
    - e.g., job scheduling, variables are start/end days for each job
    - need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$

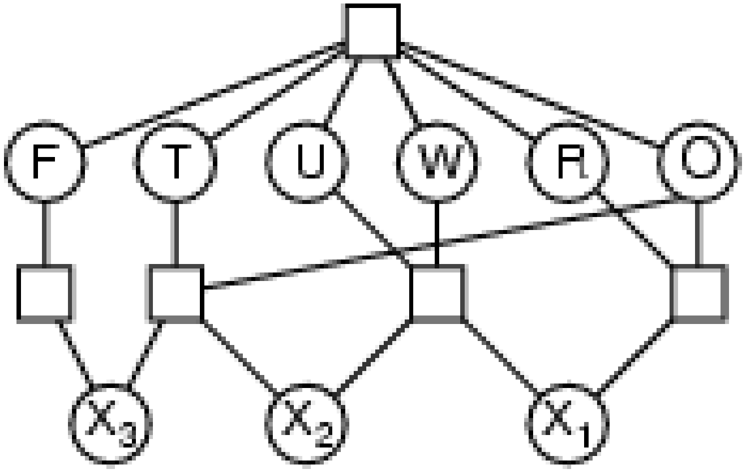
- Continuous variables
  - e.g., start/end times for Hubble Space Telescope observations
  - linear constraints solvable in polynomial time by linear programming

Varieties of constraints

- Unary constraints involve a single variable,
  - e.g., SA ≠ green
- Binary constraints involve pairs of variables,
  - e.g., SA ≠ WA
- Higher-order constraints involve 3 or more variables,
  - e.g., cryptarithmetic column constraints

Example: Cryptarithmetic

  T W O
+ T W O
F O U R



- Variables: $F\ T\ U\ W$
  $R\ O\ X_1\ X_2\ X_3$
- Domains: $\{0,1,2,3,4,5,6,7,8,9\}$
- Constraints: *Alldiff (F,T,U,W,R,O)*
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$

Real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling

- Notice that many real-world problems involve real-valued variables
- Constraints may be preferred constraints rather than absolute

Standard search formulation (incremental)

Let's start with the straightforward approach, then fix it

States are defined by the values assigned so far

- Initial state: the empty assignment { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment
  → fail if no legal assignments
- Goal test: the current assignment is complete

1. This is the same for all CSPs
2. Every solution appears at depth *n* with *n* variables
   → use depth-first search
3. Path is irrelevant, so can also use complete-state formulation
4. $b = (n - \ell)d$ at depth ℓ, hence $n! \cdot d^n$ leaves
5. But only d^n complete assignments!

Backtracking search

- Variable assignments are commutative}, i.e.,

[WA = red then NT = green] same as [NT = green then WA = red]

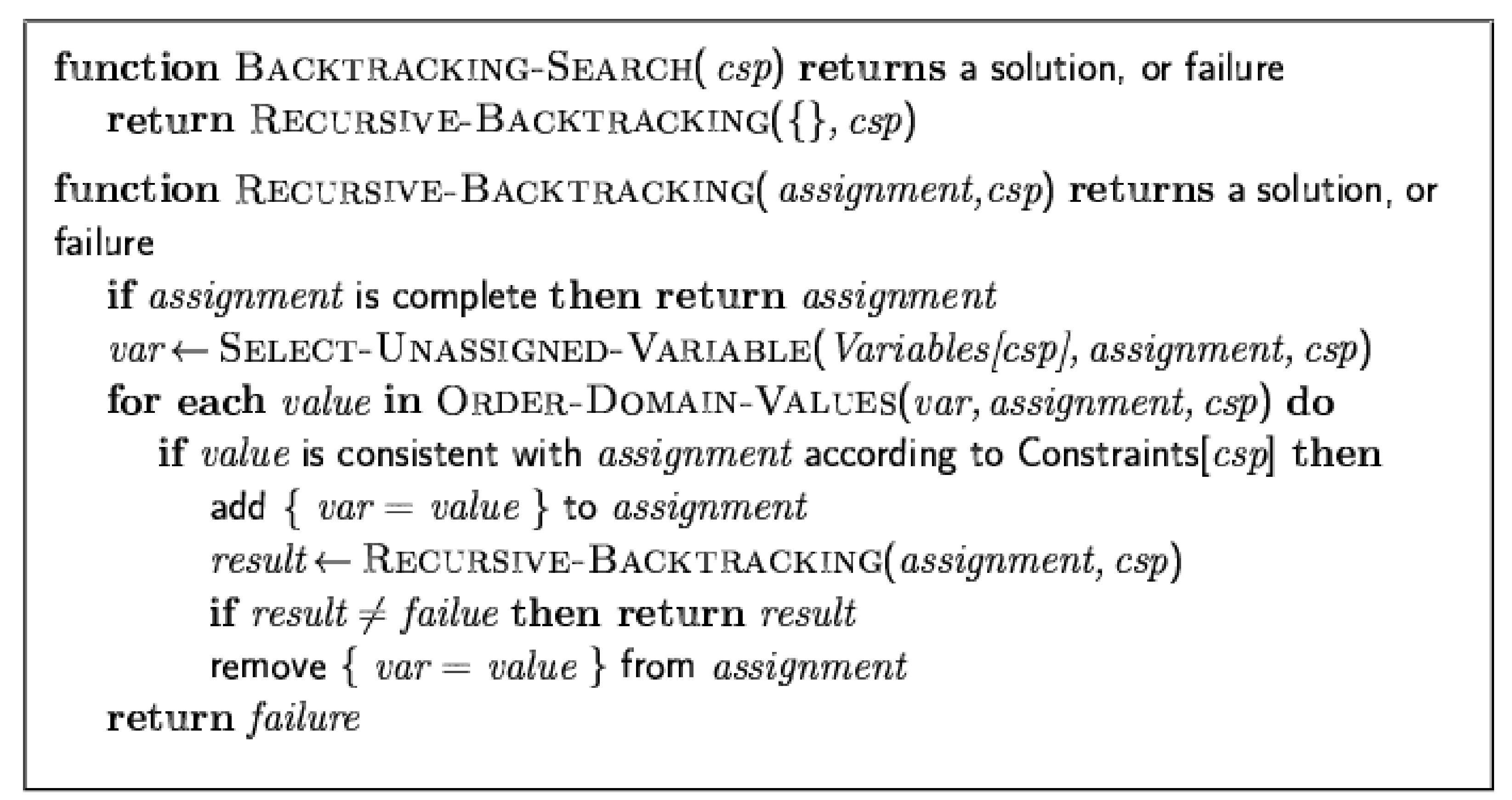
- Only need to consider assignments to a single variable at each node
  → b = d and there are \$d^n\$ leaves

- Depth-first search for CSPs with single-variable assignments is called backtracking search

- Backtracking search is the basic uninformed algorithm for CSPs

- Can solve *n*-queens for *n* ≈ 25

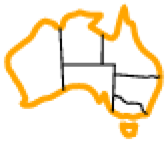
Backtracking search



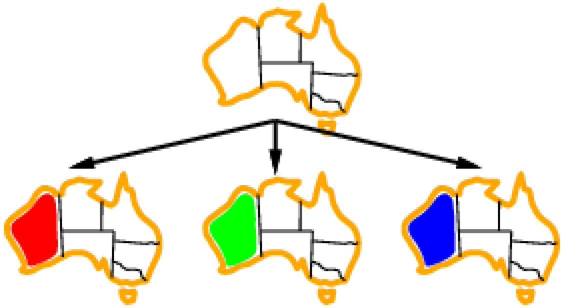
```
function BACKTRACKING-SEARCH( csp) returns a solution, or failure
    return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING( assignment, csp) returns a solution, or failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE( Variables[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment according to Constraints[csp] then
            add { var = value } to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failue then return result
            remove { var = value } from assignment
    return failure
```

Backtracking example



Backtracking example



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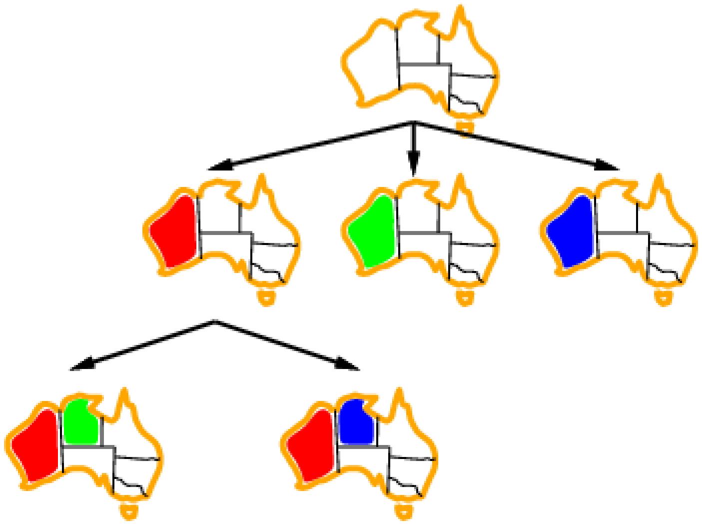

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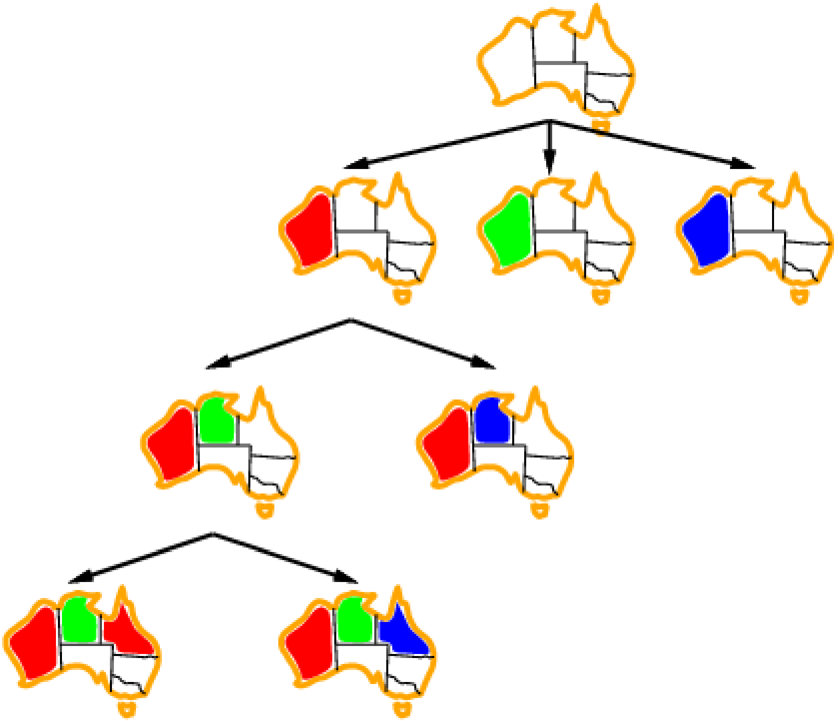
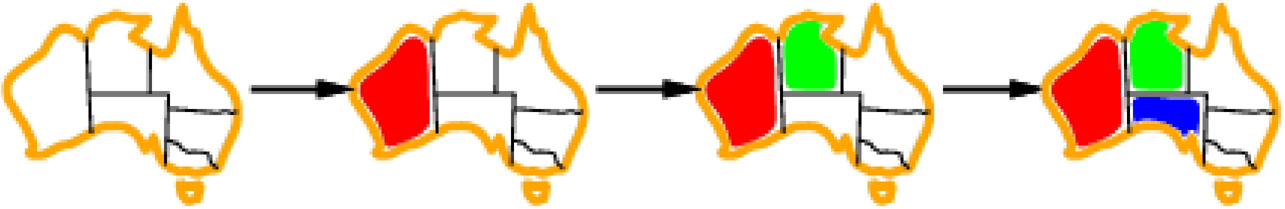


Improving backtracking efficiency

- General-purpose methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Most constrained variable

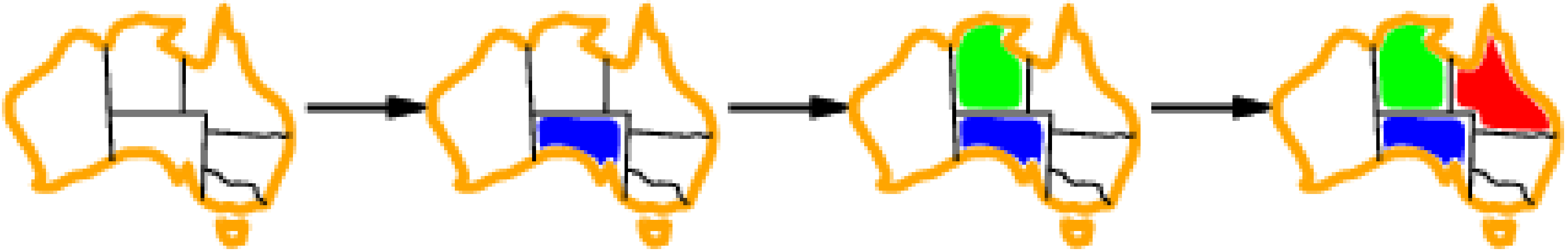
- Most constrained variable:
  choose the variable with the fewest legal values



- a.k.a. minimum remaining values (MRV) heuristic or fail first
- Magnitude of 3 to 3000 times faster than BT

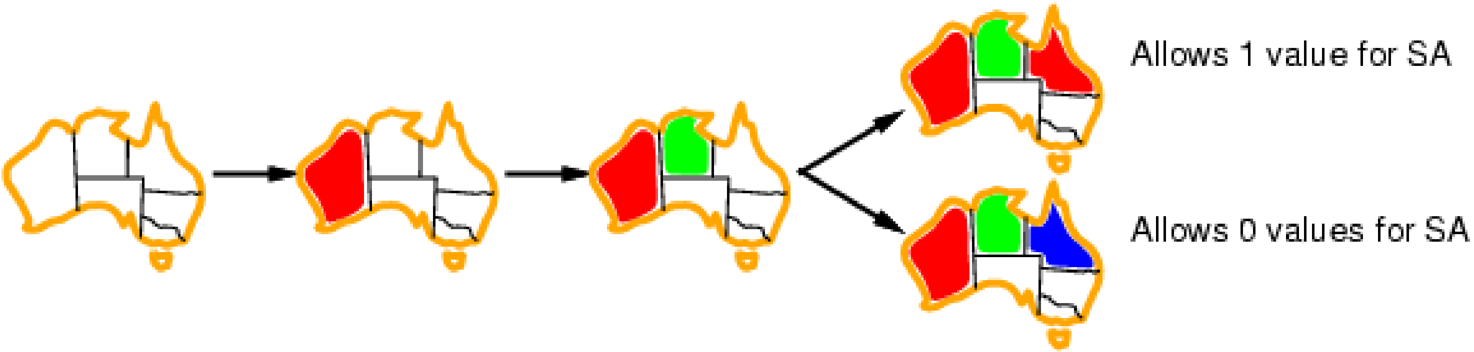
Most constraining variable

- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables



Least constraining value

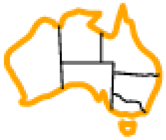
- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables



- Combining these heuristics makes 1000 queens feasible

Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦

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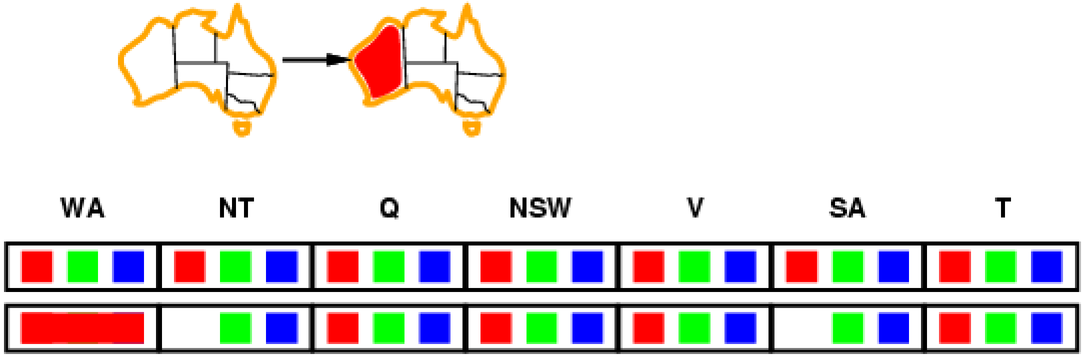
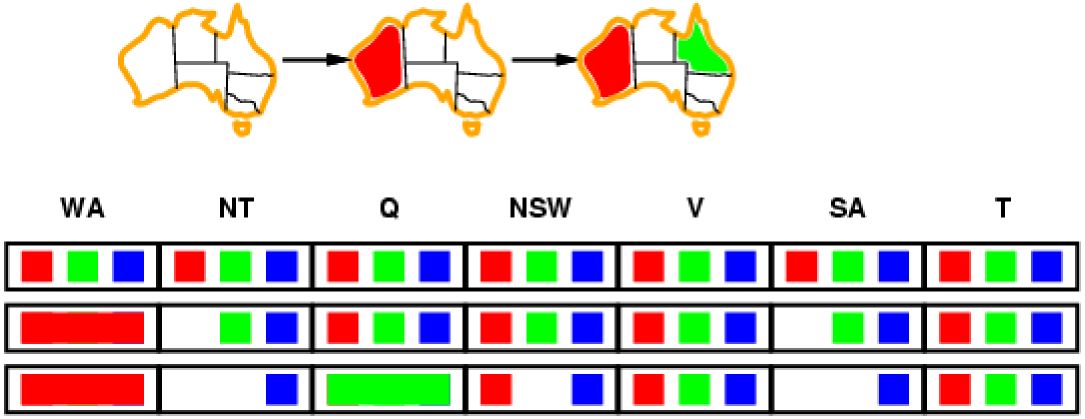


WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦
🟥	🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟩🟦	🟥🟩🟦

Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



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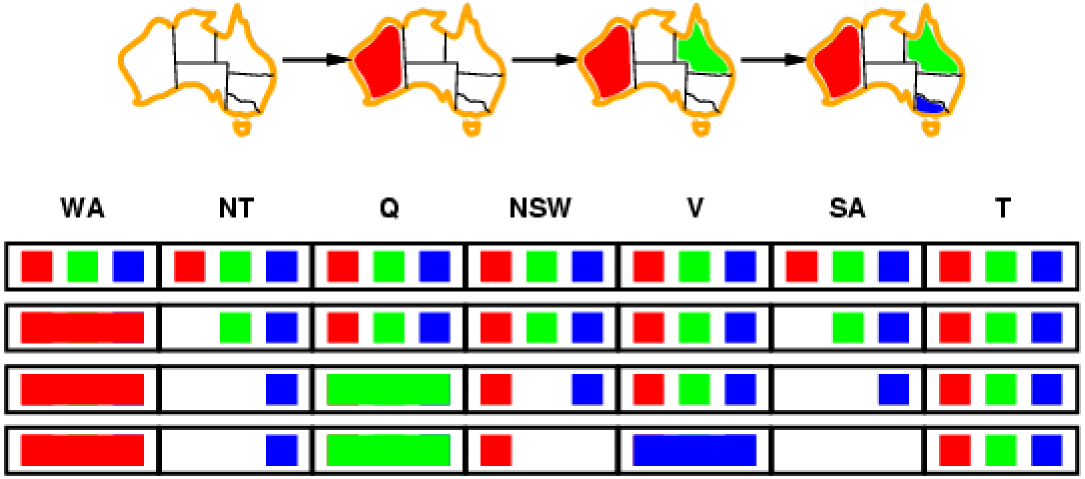

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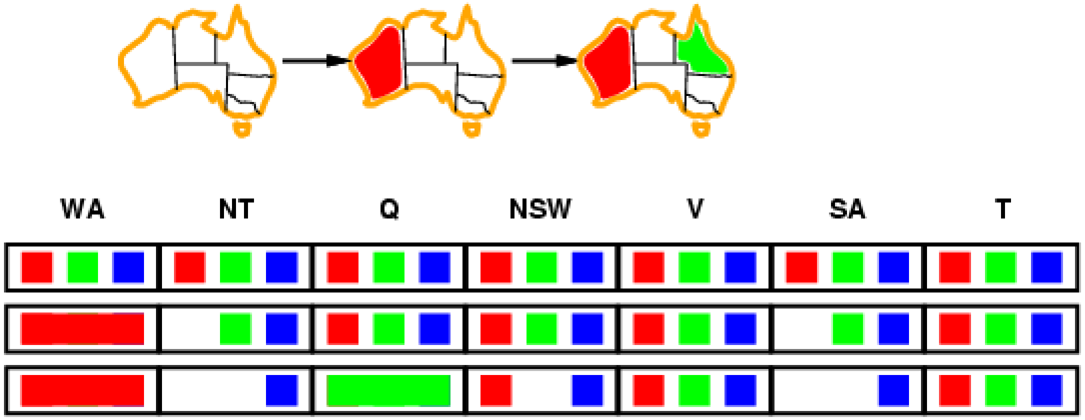
Constraint propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:



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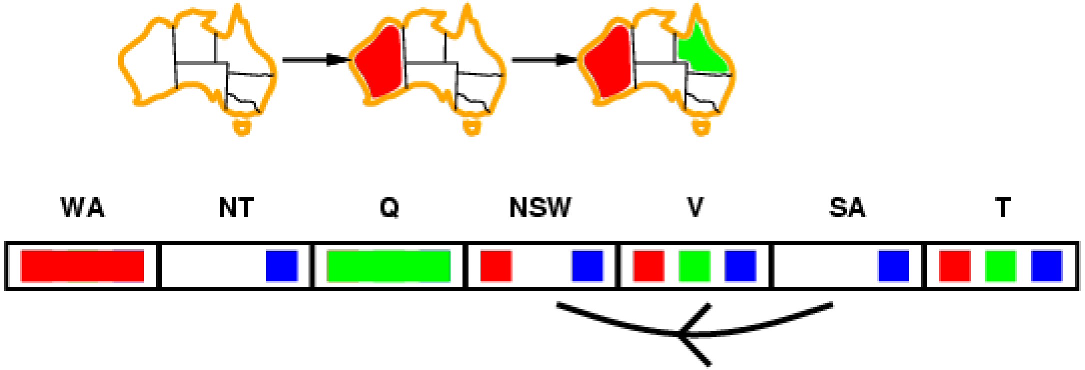
🟥	🟦	🟩	🟥🟦	🟥🟩🟦	🟦	🟥🟩🟦

- NT and SA cannot both be blue!
- Constraint propagation repeatedly enforces constraints locally

Arc consistency

- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent if

  for every value *x* of *X* there is some allowed *y*



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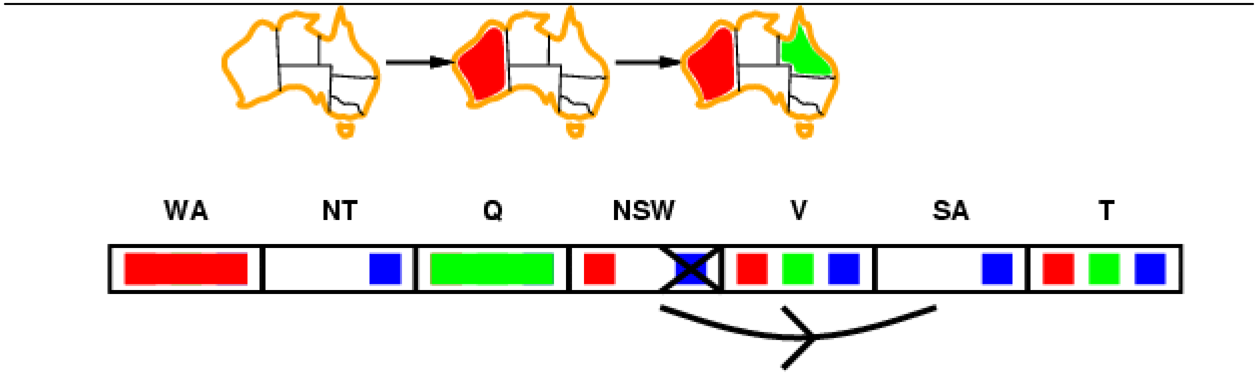

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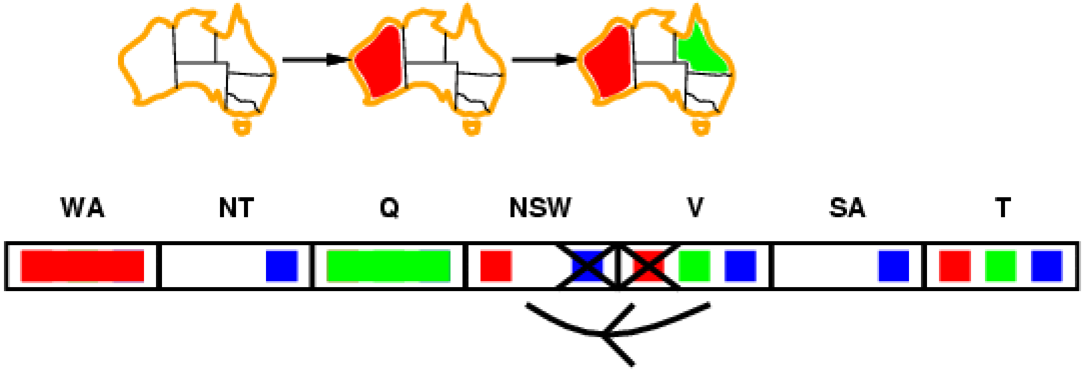

- If X loses a value, neighbors of X need to be rechecked

Arc consistency

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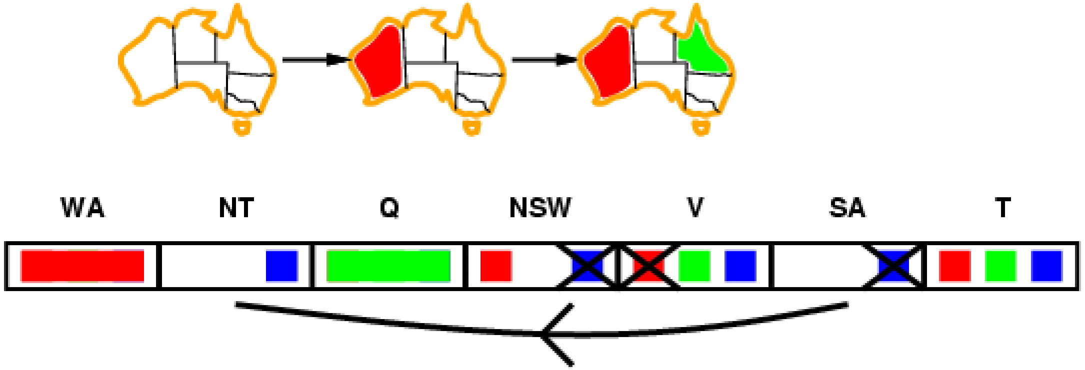
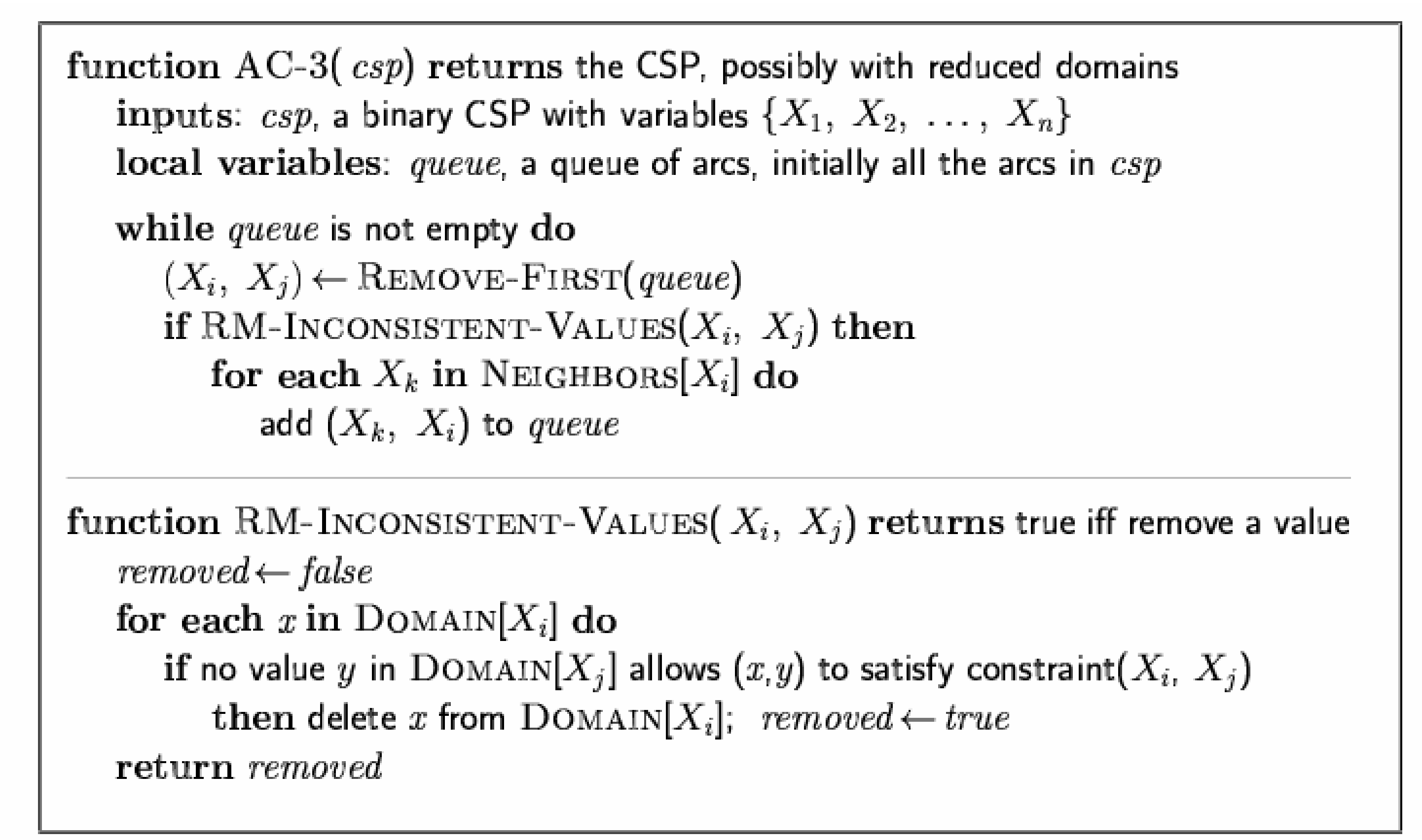


- If *X* loses a value, neighbors of *X* need to be rechecked
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment

Arc consistency algorithm AC-3



```
function AC-3( csp) returns the CSP, possibly with reduced domains
    inputs: csp, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
    local variables: queue, a queue of arcs, initially all the arcs in csp

    while queue is not empty do
        $(X_i, X_j) \leftarrow$ REMOVE-FIRST(queue)
        if RM-INCONSISTENT-VALUES($X_i$, $X_j$) then
            for each $X_k$ in NEIGHBORS[$X_i$] do
                add ($X_k$, $X_i$) to queue

function RM-INCONSISTENT-VALUES( $X_i$, $X_j$) returns true iff remove a value
    removed $\leftarrow$ false
    for each $x$ in DOMAIN[$X_i$] do
        if no value $y$ in DOMAIN[$X_j$] allows ($x$,$y$) to satisfy constraint($X_i$, $X_j$)
            then delete $x$ from DOMAIN[$X_i$];  removed $\leftarrow$ true
    return removed
```

- Time complexity: $O(n^2d^3)$

Local search for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned

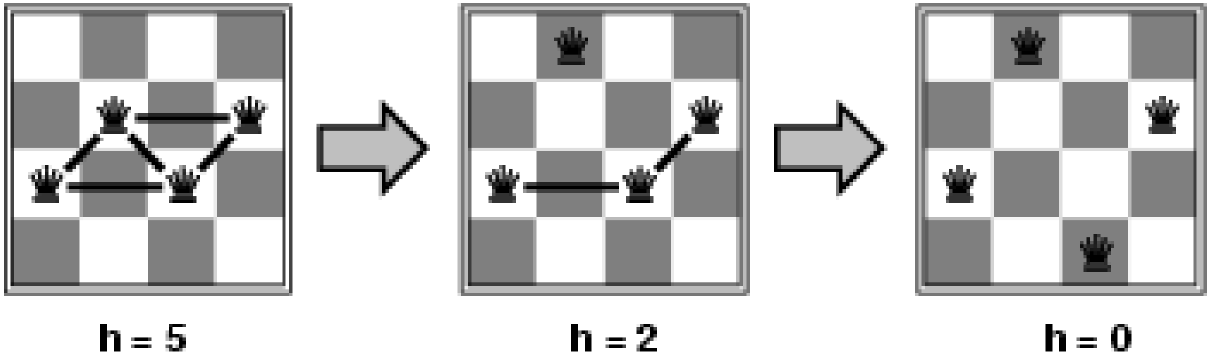
- To apply to CSPs:
  - allow states with unsatisfied constraints
  - operators reassign variable values

- Variable selection: randomly select any conflicted variable

- Value selection by min-conflicts heuristic:
  - choose value that violates the fewest constraints
  - i.e., hill-climb with *h(n)* = total number of violated constraints

Example: 4-Queens

- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Actions: move queen in column
- Goal test: no attacks
- Evaluation: *h(n)* = number of attacks



- Given random initial state, can solve *n*-queens in almost constant time for arbitrary *n* with high probability (e.g., *n* = 10,000,000)

Summary

- CSPs are a special kind of problem:
  - states defined by values of a fixed set of variables
  - goal test defined by constraints on variable values
- Backtracking = depth-first search with one variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- Iterative min-conflicts is usually effective in practice

Literatur, Links

- Stuart Russell und Peter Norvig: Künstliche Intelligenz, ein moderner Ansatz, Prentice Hall (2004), München, ISBN 3-8273-7089-2
- (daraus auch wesentliche Teile der heutigen Vorlesung)
- http://www.cs.rmit.edu.au/AI-Search/Product/
- http://aima.cs.berkeley.edu/newchap05.pdf

Wissenschaftliches Experiment zu (il)legalem P2P

- Testpersonen gesucht: Untersuchung einer Idee für eine legale Online-Musiktauschbörse
- Es gibt etwas zu essen und zu trinken! :-)
- Gruppen zu je ca. 25 Personen können nach einer Einweisung auf einer Beispiel-Website beliebig Songs herunterladen und tauschen
- Kleiner Fragebogen vor und nach dem Experiment
- Termin: Mittwoch, **7.12.2005**, fünf Termine von 10.00 bis 17.00 Uhr, Dauer ca. 1 Stunde

Anmeldung: www.intermedia.lmu.de/experiment