Themen heute

• Suchverfahren
  – Hillclimbing
  – Simulated Annealing
  – Genetische Suche

• Constraints
  – Formalisierung
  – Lösungsverfahren
Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution.

- State space = set of "complete" configurations.
- Find configuration satisfying constraints, e.g., n-queens.

- In such cases, we can use local search algorithms.
- keep a single "current" state, try to improve it.
Example: \( n \)-queens

- Put \( n \) queens on an \( n \times n \) board with no two queens on the same row, column, or diagonal

\[
\text{Hill-climbing search}
\]
- "Like climbing Everest in thick fog with amnesia"

```python
function HILL-CLIMBING(problem) returns a state that is a local maximum
inputs: problem, a problem
local variables: current, a node
neighbor, a node

current ← MAKE-NODE(INITIAL-STATE[problem])
loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
    current ← neighbor
```
Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

Hill-climbing search: 8-queens problem

- $h = \text{number of pairs of queens that are attacking each other, either directly or indirectly}$
- $h = 17$ for the above state
Hill-climbing search: 8-queens problem

- A local minimum with $h = 1$

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- The algorithm employs a random search which not only accepts changes that decrease objective function $f$, but also some changes that increase it. The latter are accepted with a probability $p = \exp\left(-\frac{\delta f}{T}\right)$
Properties of simulated annealing search

• One can prove: If $T$ decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

• Widely used in VLSI layout, airline scheduling, etc.
• Adaptation of values for $T$ is application driven.

Local beam search

• Keep track of $k$ states rather than just one

• Start with $k$ randomly generated states

• At each iteration, all the successors of all $k$ states are generated

• If any one is a goal state, stop; else select the $k$ best successors from the complete list and repeat.
Genetic algorithms

- A successor state is generated by combining two parent states

- Start with $k$ randomly generated states (population)

- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)

- Evaluation function (fitness function). Higher values for better states.

- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms

- Fitness function: number of non-attacking pairs of queens (min = 0, max = $8 \times 7/2 = 28$)

- $24/(24+23+20+11) = 31\%$

- $23/(24+23+20+11) = 29\%$ etc
Genetic algorithms

Constraint Satisfaction Problems

- Constraint Satisfaction Problems (CSP)
- Backtracking search for CSPs
- Local search for CSPs
Constraint satisfaction problems (CSPs)

- Standard search problem:
  - state is a "black box" — any data structure that supports successor function, heuristic function, and goal test
- CSP:
  - state is defined by variables $X_i$ with values from domain $D_i$
  - goal test is a set of constraints specifying allowable combinations of values for subsets of variables

- Simple example of a formal representation language

- Allows useful general-purpose algorithms with more power than standard search algorithms

Example: Map-Coloring

- Variables $WA$, $NT$, $Q$, $NSW$, $V$, $SA$, $T$
- Domains $D_i = \{\text{red, green, blue}\}$
- Constraints: adjacent regions must have different colors
  - e.g., $WA \neq NT$, or $(WA,NT)$ in
    $\{(\text{red,green}), (\text{red,blue}), (\text{green,red}), (\text{green,blue}), (\text{blue,red}), (\text{blue,green})\}$
**Example: Map-Coloring**

- **Solutions** are complete and consistent assignments, e.g., WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green

**Constraint graph**

- **Binary CSP**: each constraint relates two variables
- **Constraint graph**: nodes are variables, arcs are constraints
Varieties of CSPs

• Discrete variables
  – finite domains:
    • \( n \) variables, domain size \( d \) \( \Rightarrow \) \( O(d^n) \) complete assignments
    • e.g., Boolean CSPs, incl. Boolean satisfiability (NP-complete)
  – infinite domains:
    • integers, strings, etc.
    • e.g., job scheduling, variables are start/end days for each job
    • need a constraint language, e.g., \( \text{StartJob}_1 + 5 \leq \text{StartJob}_3 \)

• Continuous variables
  – e.g., start/end times for Hubble Space Telescope observations
  – linear constraints solvable in polynomial time by linear programming

Varieties of constraints

• **Unary** constraints involve a single variable,
  – e.g., \( \text{SA} \neq \text{green} \)

• **Binary** constraints involve pairs of variables,
  – e.g., \( \text{SA} \neq \text{WA} \)

• **Higher-order** constraints involve 3 or more variables,
  – e.g., cryptarithmetic column constraints
Example: Cryptarithmetic

Variables: \( F, T, U, W \), \( R, O, X_1, X_2, X_3 \)

Domains: \( \{0,1,2,3,4,5,6,7,8,9\} \)

Constraints:

\[
\begin{align*}
\text{Alldiff} & (F, T, U, W, R, O) \\
O + O &= R + 10 \cdot X_1 \\
X_1 + W + W &= U + 10 \cdot X_2 \\
X_2 + T + T &= O + 10 \cdot X_3 \\
X_3 &= F, \ T \neq 0, \ F \neq 0
\end{align*}
\]

Real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling

Notice that many real-world problems involve real-valued variables

Constraints may be preferred constraints rather than absolute
Standard search formulation (incremental)

Let's start with the straightforward approach, then fix it

States are defined by the values assigned so far

- **Initial state:** the empty assignment \( \{ \} \)
- **Successor function:** assign a value to an unassigned variable that does not conflict with current assignment
  - fail if no legal assignments
- **Goal test:** the current assignment is complete

1. This is the same for all CSPs
2. Every solution appears at depth \( n \) with \( n \) variables
   - use depth-first search
3. Path is irrelevant, so can also use complete-state formulation
4. \( b = (n - l)d \) at depth \( l \), hence \( n! \cdot d^n \) leaves
5. But only \( d^n \) complete assignments!

Backtracking search

- Variable assignments are **commutative**, i.e.,
  \[ \text{WA = red then NT = green} \] same as \[ \text{NT = green then WA = red} \]

- Only need to consider assignments to a single variable at each node
  - \( b = d \) and there are \( d^n \) leaves

- Depth-first search for CSPs with single-variable assignments is called **backtracking search**

- Backtracking search is the basic uninformed algorithm for CSPs

- Can solve \( n \)-queens for \( n = 25 \)
Backtracking search

```plaintext
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
    return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(Variables[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment according to Constraints[csp] then
            add { var = value } to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove { var = value } from assignment
        return failure
```

Backtracking example

![Backtracking example](image_url)
Backtracking example
Backtracking example

Improving backtracking efficiency

- **General-purpose** methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?
Most constrained variable

- Most constrained variable: choose the variable with the fewest legal values

  - a.k.a. minimum remaining values (MRV) heuristic or fail first
  - Magnitude of 3 to 3000 times faster than BT

Most constraining variable

- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables
**Least constraining value**

- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables

- Combining these heuristics makes 1000 queens feasible

**Forward checking**

- **Idea:**
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values
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Forward checking

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Constraint propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

- NT and SA cannot both be blue!
- **Constraint propagation** repeatedly enforces constraints locally
Arc consistency

- Simplest form of propagation makes each arc consistent
- $X \rightarrow Y$ is consistent if for every value $x$ of $X$ there is some allowed $y$
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- If $X$ loses a value, neighbors of $X$ need to be rechecked

Arc consistency

- Simplest form of propagation makes each arc consistent
- $X \rightarrow Y$ is consistent if
  
  for every value $x$ of $X$ there is some allowed $y$

- If $X$ loses a value, neighbors of $X$ need to be rechecked
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
Arc consistency algorithm AC-3

```
function AC-3(csp) returns the CSP, possibly with reduced domains
inputs: csp, a binary CSP with variables \{X_1, X_2, ..., X_n\}
local variables: queue, a queue of arcs, initially all the arcs in csp
while queue is not empty do
    (X_i, X_j) ← Remove-First(queue)
    if RM-INCONSISTENT-VALUES(X_i, X_j) then
        for each X_k in Neighbors(X_i) do
            add (X_k, X_i) to queue

function RM-INCONSISTENT-VALUES(X_i, X_j) returns true if a value
removed ← false:
    for each x in Domain[X_i] do
        if no value y in Domain[X_j] allows (x, y) to satisfy constraint(X_i, X_j)
            then delete x from Domain[X_i]; removed ← true
    return removed
```

- Time complexity: \(O(n^2d^3)\)

Local search for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned

- To apply to CSPs:
  - allow states with unsatisfied constraints
  - operators \textit{reassign} variable values

- Variable selection: randomly select any conflicted variable

- Value selection by \textit{min-conflicts} heuristic:
  - choose value that violates the fewest constraints
  - i.e., hill-climb with \(h(n)\) = total number of violated constraints
Example: 4-Queens

- **States**: 4 queens in 4 columns (4^4 = 256 states)
- **Actions**: move queen in column
- **Goal test**: no attacks
- **Evaluation**: \( h(n) = \) number of attacks

Given random initial state, can solve \( n \)-queens in almost constant time for arbitrary \( n \) with high probability (e.g., \( n = 10,000,000 \))

Summary

- CSPs are a special kind of problem:
  - states defined by values of a fixed set of variables
  - goal test defined by constraints on variable values
- Backtracking = depth-first search with one variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- Iterative min-conflicts is usually effective in practice
Literatur, Links

• Stuart Russell und Peter Norvig:
  Künstliche Intelligenz, ein moderner
  Ansatz, Prentice Hall (2004), München,
  ISBN 3-8273-7089-2
  (daraus auch wesentliche Teile der heutigen Vorlesung)
• http://www.cs.rmit.edu.au/Al-Search/Product/
• http://aima.cs.berkeley.edu/newchap05.pdf