Chapter 3 - 3D Modeling

- Polygon Meshes
- Geometric Primitives
- Interpolation Curves
- Levels Of Detail (LOD)
- Constructive Solid Geometry (CSG)
- Extrusion & Rotation
- Volume- and Point-based Graphics
The 3D rendering pipeline (our version for this class)

- 3D models in model coordinates
- 3D models in world coordinates
- 2D Polygons in camera coordinates
- Pixels in image coordinates

- Scene graph
- Camera
- Rasterization
- Animation, Interaction
- Lights
Representations of (Solid) 3D Objects

• Complex 3D objects need to be constructed from a set of primitives
  – Representation schema is a mapping of 3D objects --> primitives
  – Primitives should be efficiently supported by graphics hardware

• Desirable properties of representation schemata:
  – Representative power: Can represent many (or all) possible 3D objects
  – Representation is a mapping: Unique representation for any 3D object
  – Representation mapping is injective: Represented 3D object is unique
  – Representation mapping is surjective: Each possible representation value is valid
  – Representation is precise, does not make use of approximations
  – Representation is compact in terms of storage space
  – Representation enables simple algorithms for manipulation and rendering

• Most popular on modern graphics hardware:
  – Boundary representations (B-Reps) using vertices, edges and faces.
Polygon Meshes

- Describe the surface of an object as a set of polygons
- Mostly use triangles, since they are trivially convex and flat
- Current graphics hardware is optimized for triangle meshes

vertices  edge  faces  polygons  surfaces

3D Polygons and Planes

- A polygon in 3D space should be *flat*, i.e. all vertices in one 2D plane
  - Trivially fulfilled for triangles

- Mathematical descriptions of a 2D plane in 3D space (hyperplane)
  - Method 1: Point $p$ and two non-parallel vectors $v$ and $w$
    \[ x = p + sv + tw \]
  - Method 2: Three non-collinear points
    (take one point and the difference vectors to the other two)
  - Method 3: Point $p$ and normal vector $n$ for the plane
    \[ n \cdot (x - p) = 0 \] using the dot product
  - Method 4: Single plane equation
    \[ Ax_1 + Bx_2 + Cx_3 + D = 0 \] $A, B, C, D$ real numbers
    \[ (A, B, C) \] is the normal vector of the plane

- All description methods easily convertible from one to the other
  (E.g. using cross product to compute normal vector)
Example: Triangle and Associated Plane

Three points (corners of the unit cube)

\[ p = (1,0,0) \]
\[ q = (1,1,0) \]
\[ r = (1,0,1) \]

Two in-plane vectors:

\[ \vec{v} = q - p = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \vec{w} = r - p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \]

Normal vector:

\[ \vec{v} \times \vec{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix} = \begin{pmatrix} 1 - 0 \\ 0 - 0 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

Plane equation:

\[ \begin{align*}
1x_1 + 0x_2 + 0x_3 + D &= 0 \\
\text{Inserting any point gives:} \\
x_1 - 1 &= 0
\end{align*} \]
Example (contd.): Triangle Front and Back Face

Three points (corners of the unit cube)

- \( p = (1,0,0) \)
- \( q = (1,0,1) \)
- \( r = (1,1,0) \)

Different order of vertices: mathematically negative (clockwise)

Normal vector:

\[
\vec{v} \times \vec{w} = \begin{pmatrix}
-1 \\
0 \\
0
\end{pmatrix}
\]

Plane equation:

\[
(-1)x_1 + 0x_2 + 0x_3 + D = 0
\]

Inserting any point gives:

\[-x_1 + 1 = 0\]

For an arbitrary point:
Left hand side of plane equation gives value > 0: Point is in front of plane
Left hand side of plane equation gives value < 0: Point is behind plane
Right Hand Rule for Polygons

• A “rule of thumb” to determine the front side (= direction of the normal vector) for a polygon
• Please note: The relationship between vertex order and normal vector is just a convention!
  – Q: How can we see this from the previous slides?

Source: http://www.csse.monash.edu.au/~cema
Face-Vertex Meshes

Möbius Strip: Non-Orientable Surface

Complete object: Does not have a front and back side!

M. C. Escher: Moebius Strip II
Polygon Meshes: Optional data

- Color per vertex or per face: produces colored models
- Normal per face:
  - Easy access to front/back information (for visibility tests)
- Normal per vertex:
  - Standard computation accelerated (average of face normals)
  - Allows free control over the normals
    - use weighted averages of normals
    - mix smooth and sharp edges (VRML/X3D: crease angles)
    - wait for shading chapter ;-)
- Texture coordinates per vertex
  - wait for texture chapter ;-)
Polygon Meshes: other descriptions

- Other representations for polygon meshes exist
  - optimized for analyzing and modifying topology
  - optimized for accessing large models
  - optimized for fast rendering algorithms
  - optimized for graphics hardware

- Example: triangle strip
  - needs N+2 points for N polygons
  - implicit definition of the triangles
  - optimized on graphics hardware
  - OpenGL / JOGL:
    ```
    gl.glBegin(GL2.GL_TRIANGLE_STRIP);
    gl.glVertex3d(-1, -1, 1);
    ...  
    ```

Approximating Primitives by Polygon Meshes

• Trivial for non-curved primitives...
• The curved surface of a cylinder, sphere etc. must be represented by polygons somehow (Tesselation).
• Not trivial, only an approximation and certainly not unique!
  – GLU utility functions for tesselation exist
• Goal: small polygons for strong curvature, larger ones for areas of weak curvature
  – This means ideally constant polygon size for a sphere
  – Where do we know this problem from??? Something playful...

http://www.evilbastard.org/slight/tesselation.gif
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Geometric Primitives

• Simplest way to describe geometric objects
• Can be used directly by some renderers (e.g., Ray tracing)
• Can be transformed into polygons easily (Tesselation)
• Can be transformed into Voxels easily
• Useful for creating simple block world models

• Supported in many frameworks of different levels
  – VRML/X3D, Java 3D
  – OpenGL, WebGL, JOGL
Box

- Described by (width, length, height)
- Origin usually in the center
- 8 points, 12 edges, 6 rectangles, 12 triangles
Pyramid, Tetrahedron (*Tetraeder*)

- Basis of pyramid = rectangle
- given by (width, length, height)
- 5 points, 8 edges, 6 triangles

- Basis of tetrahedron = triangle
- given by (width, length, height)
- 4 points, 6 edges, 4 triangles,
Generalization: Polyhedra

• Polyhedron (*Polyeder*):
  – Graphical object where a set of surface *polygons* separates the interior from the exterior
  – Most frequently used and best supported by hardware: surface triangles
  – Representation: Table of
    • Vertex coordinates
    • Additional information, like surface normal vector for polygons

• Regular polyhedra: Five Platonic regular polyhedra exist
  – Tetrahedron (*Tetraeder*)
  – Hexahedron, Cube (*Hexaeder, Würfel*)
  – Oktahedron (*Oktaeder*)
  – Dodekahedron (*Dodekaeder*)
  – Icosahedron (*Ikosaeder*)

http://www.aleakybos.ch/
Cylinder, cone, truncated cone

- Cylinder given by (radius, height)
- Number of polygons dep. on tesselation

- Cone given by (radius, height)
- Number of polygons dep. on tesselation

- Truncated cone given by (r1, r2, height)
- Number of polygons dep. on tesselation

- Q: Which of these would you rather have if you only had one available?
Sphere, Torus

- Sphere is described by (radius)
- Torus is defined by (radius1, radius2)
- Number of polygons dep. on tessellation
Geometric Primitives: Summary

- Not all of these exist in all graphics packages
- Some packages define additional primitives (dodecahedron, teapot...;-)

- Practically the only way to model in a text editor
- Can give quite accurate models
- Extremely lean! Very few polygons

- Think of application areas even in times of powerful PC graphics cards!
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**Interpolation Curves**

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Interpolation Curves, Splines

- Original idea: „Spline“ used in ship construction to build smooth shapes:
  - Elastic wooden band
  - Fixed in certain positions and directions
  - Mathematically simulated by interpolation curves
  - Piecewise described by polynomials

- Different types exist
  - Natural splines
  - Bézier curves
  - B-Splines

- Control points may be on the line or outside of it.
  - All on the line for a natural spline
Bézier Curves (and de-Casteljau Algorithm)

- Bézier curves first used in automobile construction (1960s, Pierre Bézier – Renault, Paul de Casteljau – Citroën)
- Degree 1: straight line interpolated between 2 points
- Degree 2: quadratic polynomial
- Degree 3: cubic Bézier curve, described by cubic polynomial
- Curve is always contained in convex hull of points
- Algorithm (defines line recursively):
  - Choose $t$ between 0 and 1
  - I1: Divide line between P1 and P2 as $t : (1–t)$
  - I2, I3: Repeat for all Ps (one segment less!)
  - J1, J2: Repeat for I1, I2, I3 (same $t$)
  - K: Repeat for J1, J2 (single point!)
  - Bézier curve: all points K for $t$ between 0 and 1
- see http://goo.gl/m7Z1Y (Dominik Menke)
Bézier Patches

• Combine 4 Bézier curves along 2 axes
• Share 16 control points
• Results in a smooth surface
• Entire surface is always contained within the convex hull of all control points
• Border line is fully determined by border control points
• Several patches can be combined
  – connect perfectly if border control points are the same.
• Advantage: move just one control point to deform a larger surface...
• Other interpolation surfaces based on other curves
  – Generalization of Bézier idea: B-splines
  – Further generalization: Non-uniform B-splines
  – Non-uniform rational B-splines (NURBS) (supported by OpenGL GLU)
Interpolation in OpenGL (Bezier Example)

• Utah teapot
  – Martin Newell, 1975
  – 306 vertices
  – 32 bicubic Bézier surface patches

http://www.realtimerendering.com/teapot/
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Levels of Detail

• Assume you have a very detailed model
• from close distance, you need all polygons
• from a far distance, it only fills a few pixels
• How can we avoid drawing all polygons?
Mesh reduction

• Original: ~5.000 polygons
• Reduced model: ~1.000 polygons
• ==> about 80% reduction

• Very strong reductions possible, depending on initial mesh

• Loss of shape if overdone

A method for polygon reduction

- Rossignac and Borell, 1992, „Vertex clustering“
- Subdivide space into a regular 3D grid
- For each grid cell, melt all vertices into one
  - Choose center of gravity of all vertices as new one
  - Triangles within one cell disappear
  - Triangles across 2 cells become edges (i.e. disappear)
  - Triangles across 3 cells remain
- Good guess for the minimum size of a triangle
  - edge length roughly = cell size
- Yields constant vertex density in space
- Does not pay attention to curvature
Billboard

• A flat object which is always facing you
• Very cheap in terms of polygons (2 triangles)
• Needs a meaningful texture
• Example (from SketchUp): guy in the initial empty world rotates about his vertical axis to always face you
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Constructive Solid Geometry

- Basic idea: allow geometric primitives and all sorts of boolean operations for combining them
- Can build surprisingly complex objects
- Good for objects with holes (often the simplest way)
- Basic operations:
  - \textbf{Or}: combine the volume of 2 objects
  - \textbf{And}: intersect the volume of 2 objects
  - \textbf{Not}: all but the volume of an object
  - \textbf{Xor}: all space where 1 object is, but not both
- Think about:
  - wheels of this car
  - tea mug
  - coke bottle (Problems??)
CSG: a complex Example

- rounded_cube = cube And sphere

- cross = cyl1 Or cyl2 Or cyl3

- result = rounded_cube And (Not cross)

Think: Are CSG operations associative?

...commutative?
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Extrusion (sweep object)

- Move a 2D shape along an arbitrary path
- Possibly also scale in each step

http://www.cadimage.net/cadtutor/lisp/helix-02.gif
Rotation

• Rotate a 2D shape around an arbitrary axis
• Can be expressed by extrusion along a circle

• How can we model a vase?
  –
  –
  –

• How a Coke bottle?
  –
  –
  –
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Voxel data

• „Voxel“ = „Volume“ + „Pixel“, i.e., voxel = smallest unit of volume
• Regular 3D grid in space
• Each cell is either filled or not
• Memory increases (cubic) with precision

• Easily derived from CSG models
• Also the result of medical scanning devices
  – MRI, CT, 3D ultrasonic

• Volume rendering = own field of research
• Surface reconstruction from voxels

http://www.drububu.com/tutorial/voxels.html
Point-based graphics

• Objects represented by point samples of their surface („Surfels“)
• Each point has a position and a color
• Surface can be visually reconstructed from these points
  – purely image-based rendering
  – no mesh structure
  – very simple source data (x,y,z,color)

• Point-data is acquired e.g., by 3D cameras
• Own rendering techniques
• Own pipeline
• ==> own lecture ;-)

http://www.crs4.it/vic/data/images/img-exported/stmatthew_4px_full_shaded2.png