Assignment 2 - Transformations

This assignment makes you practice transformation in computer graphics. For your in-depth understanding of this area, it also covers a few related topics that might new to you. You should use any resources (e.g., books, search engines, calculators, and etc.) that can help you accomplish it.

Task 1: Affine Transformation

Let \( P' = (p_1', p_2', p_3')^\top \) transformed from \( P \) by a scaling \( T_s \) with factors \( s_x, s_y, \) and \( s_z \); then followed by a translation \( T_t \) on the direction of vector \( t = (t_x, t_y, t_z)^\top \).

a) Write the homogeneous matrices \( T_s \) and \( T_t \) these two operations.

b) Why do we need homogeneous coordinates? Name three reasons.

As we can see, the geometric meaning of a matrix is a transformation that maps a point to another. With \( T_t \) and \( T_s \), we have:

\[
P' = T_t T_s P
\]

To find the original point \( P \), one can multiply two inverse matrix \( T_s^{-1} \) and \( T_t^{-1} \) that comply:

\[
T_s^{-1} T_t^{-1} P' = T_s^{-1} T_t^{-1} T_t T_s P = P
\]

i.e. \( T_s^{-1} T_t^{-1} T_t T_s = I \) where \( I \) is the identity matrix.

c) Calculate matrices \( T_s^{-1} \) and \( T_t^{-1} \) that inverse transformations you wrote in a).

d) Calculate the coordinates of point \( P \) with respect to \( P' \).

e) Can you switch the order of the transformations? Show your evidence by comparing \( T_t T_s \) and \( T_s T_t \).

In particular, given \( Q = (x_0 + mt, y_0 + nt, z_0 + pt)^\top \) on the direction of \( v = (m,n,p)^\top \) concerning \( M_0(x_0, y_0, z_0) \) where \( t \in \mathbb{R} \):

f) Calculate \( T_t T_s Q \)

g) What property of affine transformation is showed in the question f)? Why?

Include your answers in a Markdown file called “task01.md”.

Computer Graphics, Summer semester 2020, LMU Munich
**Task 2: Rotation with Euler Angles**

Assume we use a right-handed system. In the lecture, you learned an example to perform 3D rotation, i.e. rotation by $\theta_x$ around $x$-axis. The rotation matrix is:

$$R_x = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_x & -\sin \theta_x \\
0 & \sin \theta_x & \cos \theta_x
\end{pmatrix}$$

In practice, rotation in three-dimensional space is more complicated to describe than a single axis rotation. The rotation angles $(\theta_x, \theta_y, \theta_z)$ around axes of the global (world) coordinate system is the so-called extrinsic Euler angles, and rotation angles $(\theta_X, \theta_Y, \theta_Z)$ around axes of the local (object) coordinate system (solidary with the moving body) is the so-called intrinsic Euler angles. We use extrinsic Euler angles in this task.

a) What are the rotation matrices $R_y$ and $R_z$ for Euler angles $\theta_y$ and $\theta_z$?

b) Calculate the rotation matrix $R_x R_y R_z$ that represents a series of rotations around $x$, $y$ then $z$-axis.

c) Can you switch the order of the transformations? Show your evidence by comparing $R_x R_y R_z$ and $R_z R_y R_x$.

d) Let $\theta_y = \frac{\pi}{2}$, simplify the matrix $R_x R_y R_z$ you calculated in the question b).

e) Let point $L = (l_1, l_2, l_3)^\top$, calculate $R_x R_y R_z L$ where $\theta_y = \frac{\pi}{2}$. What can you conclude from the result you have? (Hint: Think about where the point is after the transformation.)

Moreover, the rotation sequences regarding three different axes are also called *Tait-Bryan angles*. For instance: $R_z R_y R_x$. However, one can also perform the rotation 3 times with respect to only two axes, which is also called *Proper Euler angles*. For instance: $R_x R_y R_x$.

f) How many possible sequences in Tait-Bryan angles?

g) How many possible sequences in Proper Euler angles?

Include your answers in a Markdown file called “task02.md”.

**Task 3: Rotation with Quaternions**

A better and strict way of doing rotation in 3D space is using *quaternions*, which are used more common in practice. Quaternion is similar complex numbers but more abstract. A complex number $z = a + bi \in \mathbb{C}$ where $i^2 = -1$ is the *imaginary unit*, and $a, b \in \mathbb{R}$. The geometric meaning of complex number multiplication embeds a rotation. For instance, given a real number $r$ on the $x$-axis, the multiplication $ri$ means a counterclockwise rotation of $r$ by $\frac{\pi}{2}$; and the multiplication $r ii = ri^2 = -r$ means two counterclockwise rotations and each by $\frac{\pi}{2}$. Intuitively, a complex number is a point on a 2D plane.

Instead of a single imaginary unit, a quaternion $q$ contains more than one imaginary unit:
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\[ q = a + bi + cj + dk \in \mathbb{H} = \text{span}(1, i, j, k) \]

where \( i, j, k \) are imaginary units and

\[ i^2 = j^2 = k^2 = ijk = -1 \]

a) Assume \( p = e + fi + gj + hk \), calculate quaternions multiplication \( pq \) arithmetically.

Since a quaternion is represented as a real number and three imaginary parts, one can use scalar-vector pair to represent a quaternion \( q = (a, v) \) where \( v = (b, c, d) \in \mathbb{R}^3 \), and the multiplication of two quaternions can be simplified as:

\[ pq = (e, w)(a, v) = (ea - w^T \cdot v, ew + av + w \times v) \]

where \( p = (e, w) \), \( w = (f, g, h)^T \), \( \cdot \) is the dot product, and \( \times \) is the cross product. Similar to conjugate complex number \( \bar{z} = a - bi \) with respect to \( z = a + bi \), a conjugate quaternion \( \bar{q} = (a, -v) \), and the norm of a quaternion is defined as \( \|q\| = \sqrt{\bar{q}q} \).

b) Assume \( q = (\cos \theta, u \sin \theta) \), \( \theta \in \mathbb{R} \), prove \( \|q\| = 1 \) if and only if \( \|u\| = 1 \).

A rotation by angle \( \theta \) around a given unit direction \( u \) can be expressed by \( q = (\cos \frac{\theta}{2}, u \sin \frac{\theta}{2}) \). In particular, given the original point \( v \in \mathbb{R}^3 \) and the calculation \( q(0,v)\tilde{q} \) results in a quaternion \((s,v')\) where \( v' \) is the destination of the counterclockwise rotation around \( u \).

c) Let \( q_1 = (0, v), v = (0, 2, 0)^T \), \( q_2 = (\cos \frac{\pi}{4}, u \sin \frac{\pi}{4}), u = (1, 0, 0)^T \). Calculate \( q_2q_1q_2 \), \( q_2q_1 \).

d) Calculate the quaternions \( q_x, q_y, q_z \) that can express the rotations by angle \( \theta \) around \( x \)-axis, \( y \)-axis, and \( z \)-axis respectively.

Include your answers in a Markdown file called “task03.md”.

Task 4: Building A Scene using Three.js

Three.js \(^1\) is a JavaScript library that built on top of WebGL \(^2\) to create and display 3D graphics. As a first step to the graphics world, we use it for high-level constructions. This task helps you get familiar with the general coding scheme in Three.js. Your final scene should be similar to Figure 1.

Before you start working on building this scene, let’s understand how the scene is constructed. In this scene, there are three bunnies above a grid plane, as well as a right-handed Cartesian coordinate. More precisely, each bunny consists of a mesh and a material. The coordinate contains three axes, and each axis consists of a cylinder, a cone, and a text label. Moreover, a point light is located in the scene where close to the bunny on the right, and a camera that captures the whole scene. You can access an online demo \(^3\) to explore and understand details of this scene more interactively.

\(^1\) https://threejs.org
\(^2\) https://developer.mozilla.org/en-US/docs/Web/API/WebGL_API
\(^3\) http://www.medien.ifi.lmu.de/lehre/ss20/cg1/demo/2-transform/
a) What is the scene graph of this scene?

b) What nodes in your scene graph contains transformation matrices?

c) How is the position of the bunny on the right is calculated given your scene graph? Give the needed parameters and calculate the final homogeneous transformation matrix.

To start coding and run the code skeleton (https://github.com/mimuc/cg1-ss20), you need `npm i` to install all dependencies we need, then use `npm start`. The command compiles your current code skeleton and opens a new web page automatically to demonstrate your results. More conveniently, it automatically refreshes the web page while you changing your implementation.

In the src folder, you should find two JS files, and the `main.js` boots the whole runtime, creates a World object, setup the scene, then renders it:

```javascript
import World from './world'
(new World()).setupScene().render()
```

The World class includes the following key components:

- `constructor()` initializes the renderer  
- `render()` executes the render loop  
- `setupScene()` calls `setupHelpers` and `setupBunnies` where establish the whole scene we are about to create  
- `setupHelpers()` includes a helper grid plane, better visualized axes and a point light that lights up the scene
• setupBunnies loads and transforms a group of bunnies

See the code below:

```javascript
export default class World {
    constructor() {
        ... // initialize renderer
    }
    render() {
        ... // execute the render loop
    }
    setupScene() {
        this.setupHelpers()
        this.setupBunnies()
        return this
    }
    setupHelpers() {
        // creates many helpers
        this.setupGridPlane()
        this.setupAxes()
        this.setupLight()
    }
    setupBunnies() {
        // creates a group of bunnies
        const rabbits = new Group()
        const loader = new GLTFLoader()
        loader.load('assets/bunny.glb', ...) // TODO: fill in the path to the GLTF file
        this.scene.add(rabbits)
    }
    ... // more implementations
}
```

d) Look for // TODO: in the world.js, then implement them to reproduce this scene.

Hint:
- Read three.js documentation and get familiar with these concepts: Scene and PerspectiveCamera, OrbitControls, Mesh, Geometry, Material, and PointLight.
- Use the provided constant parameters in the code skeleton for better reproducibility of the scene.
- Use debugging tools (e.g. insert break points) to trace the code logic if you don’t fully understand it.

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5 https://threejs.org/docs/index.html#examples/en/controls/OrbitControls
6 https://threejs.org/docs/index.html#api/en/objects/Mesh
7 https://threejs.org/docs/index.html#api/en/core/Geometry
8 https://threejs.org/docs/index.html#api/en/materials/Material
9 https://threejs.org/docs/index.html#api/en/lights/PointLight
• Think about how to test your implementation quickly. For instance, figure out TODO should be implemented first

After reproducing this scene, you should be able to answer the following questions:

e) What are the two components that determine the final rendered scene in the render loop?

f) What are the two components that create a Mesh?

g) Where the positive Y-axis points to in three.js by default?

h) What type of Euler angles is used in three.js?

Answer text questions in the README.md of the code skeleton, then include your answers and implementation in a folder called “task04”. **Exclude** the installed dependencies (folder node_modules) in your submission.
Submission

- Participation in the exercises and submission of the weekly exercise sheets is voluntary and not a prerequisite for participation in the exam. However, participation in an exercise is a good preparation for the exam (the content is the content of the lecture and the exercise).

- For non-coding tasks, write your answers in a Markdown file. Markdown is a simple mark-up language that can be learned within a minute. A recommended the Markdown GUI parser is typora (https://typora.io/), it supports parsing embedded formula in a Markdown file. You can find the syntax reference in its Help menu.

- Please turn your solution as a ZIP file via Uni2Work (https://uni2work.ifi.lmu.de/) before the deadline. We do not accept group submissions.

- Your solution will be corrected before the discussion. Comment your code properly, organize the code well, and make sure your submission clearer can help us to give you maximum feedback.

- If we discover cheating behavior or any kind of fraud in solving the assignments, you will be withdrawn for the entire course! If that happens, you can only rejoin the course next year.

- If you have any questions, please discuss it with your fellow students first. If the problem cannot be resolved, please contact your tutorial tutor or discuss it in our Slack channel.