

# Statistics for User Studies

A Practical Approach

# Accuracy vs. Precision

## **Accuracy**

is determined by  
**measurement errors**

needed:

**good study design**

verified by:

**thorough description  
of study setup**

## **Precision**

is determined by  
**measurement noise**

needed:

**enough data**

verified by:

**rigorous statistical  
analysis**

# Types of Data

- **Categorical / Nominal Data**

(alternatives in non-overlapping subsets,  $A=B$ ,  $A \neq B$ )

- Gender: male/female, Handedness: left/right

- **Ordinal Data**

(ranking/ordering  $A > B$ ,  $A < B$ ,  $A = B$ )

- Marks in school: 1, 2, 3, 4, 5, 6
- Type of education: school, high school, university

- **Interval Scale Data**

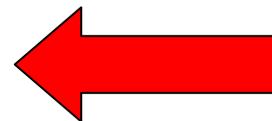
(zero point is arbitrary,  $A - B$ )

- tide
- temperature ( $^{\circ}\text{K}/^{\circ}\text{C}/^{\circ}\text{F}$ ),

- **Ratio Scale Data**

(fixed zero point  $A / B$ )

- weight
- time



**Try to get this!**

non-parametric

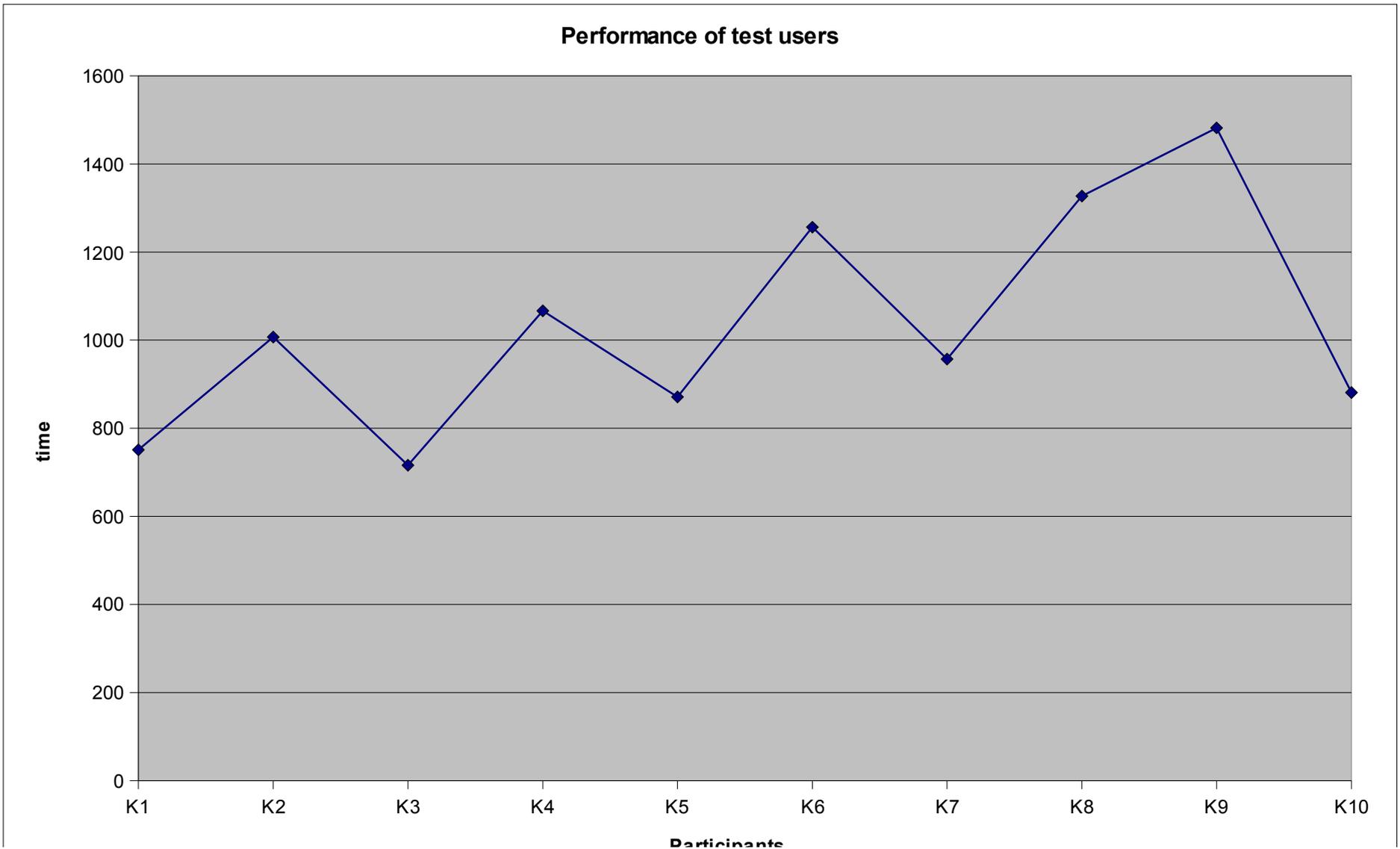
parametric

# Types of Variables

- Discrete Data
  - distinct and separate
  - can be counted
- Continuous Data
  - any value within a finite or infinite interval
  - always have an order

# Don't Do This

(in most cases)



# Frequency Tables

Data can be summarized in form of a frequency table

- well suited for discrete data
- continuous data have to be divided in groups

Example: days needed to answer my email

Data: 5 2 2 3 4 4 3 2 0 3 0 3 2 1 5 1 3 1 5 5 2 4 0 0 4 5 4 4 5 5

| <i>Days</i> | <i>Frequency</i> | <i>Frequency (%)</i> |
|-------------|------------------|----------------------|
| 0           | 4                | 13%                  |
| 1           | 3                | 10%                  |
| 2           | 5                | 17%                  |
| 3           | 5                | 17%                  |
| 4           | 6                | 20%                  |
| 5           | 7                | 23%                  |

# Mean, Median, Mode (I)

## Mean

If  $x_1, x_2, \dots, x_n$  are the data in a sample, the mean is  $\frac{1}{n} \sum_{i=1}^n X_i$

## Median

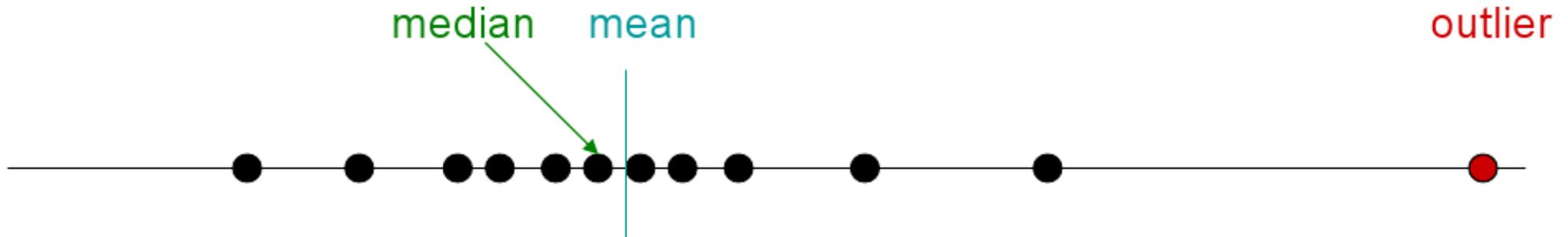
If  $x_1, x_2, \dots, x_n$  are the ordered data in a sample, the median is  $x_{(n+1)/2}$  if  $n$  is odd, and  $(x_{n/2} + x_{n/2+1}) / 2$  if  $n$  is even. It is the value halfway through the ordered data set.

## Mode

The mode is the value that occurs most often in a sample.  
There may be more than one mode in a sample.

# Mean, Median, Mode (II)

Median is less sensitive on outliers



Mode works on all types of data

Median works on ordinal, interval, ratio data

Mean works on interval or ratio data

# Likert Scales

## Examples:

PowerPoint presentations are the best way to teach. State your opinion.

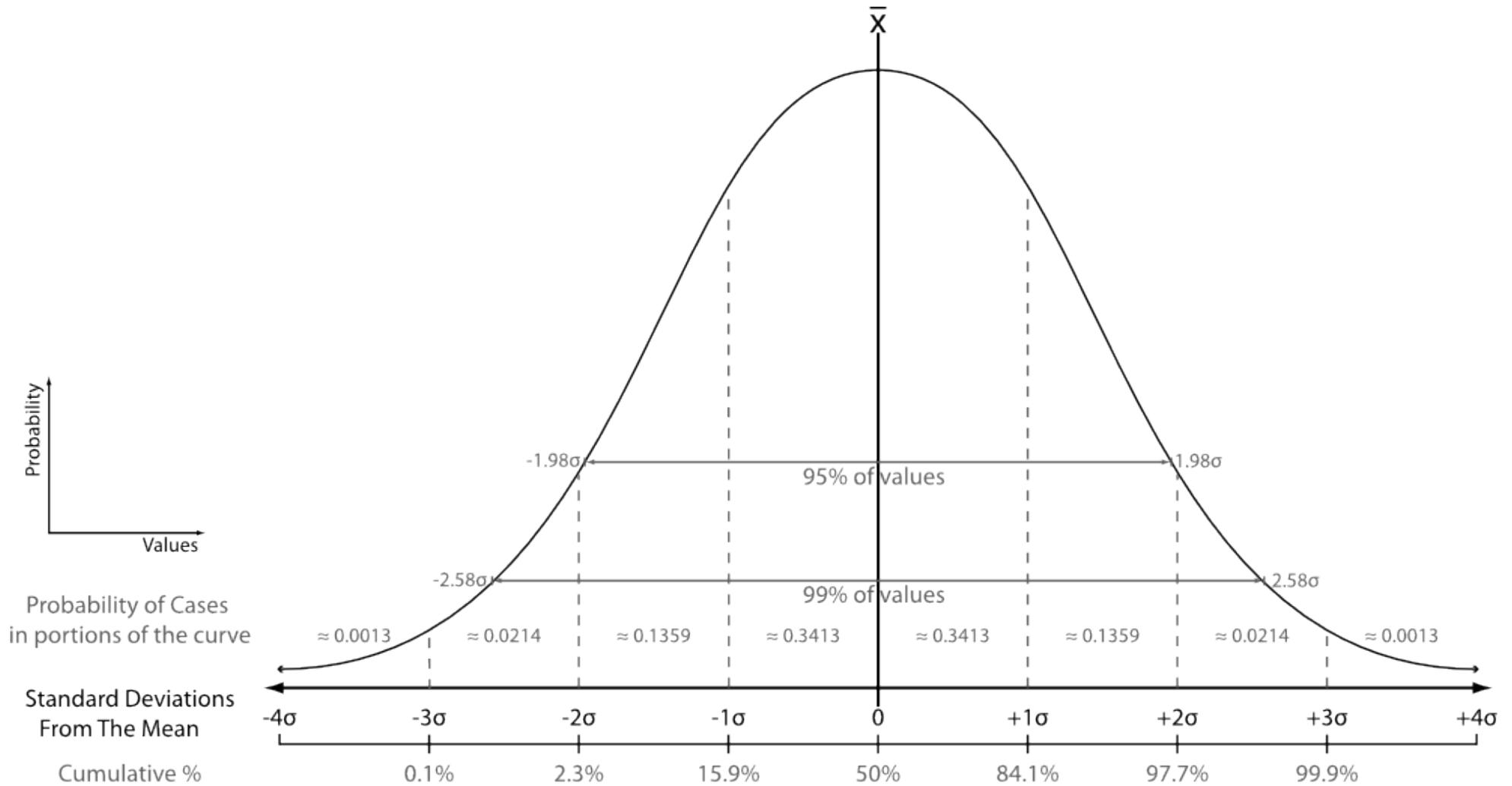
1. Strongly disagree
2. Disagree
3. Neither agree nor disagree
4. Agree
5. Strongly agree

This year I will buy a new computer.

- No       Uncertain       Yes

- ordinal data  
⇒ do not calculate mean!
- use sum-of-ranks
- you can force the user to make a commitment to one direction by offering an even number of choices.
- use 3 to 7 options

# Normal Distribution



Source: [http://en.wikipedia.org/wiki/Image:The\\_Normal\\_Distribution.svg](http://en.wikipedia.org/wiki/Image:The_Normal_Distribution.svg)

# Variance and Standard Deviation

## Variance

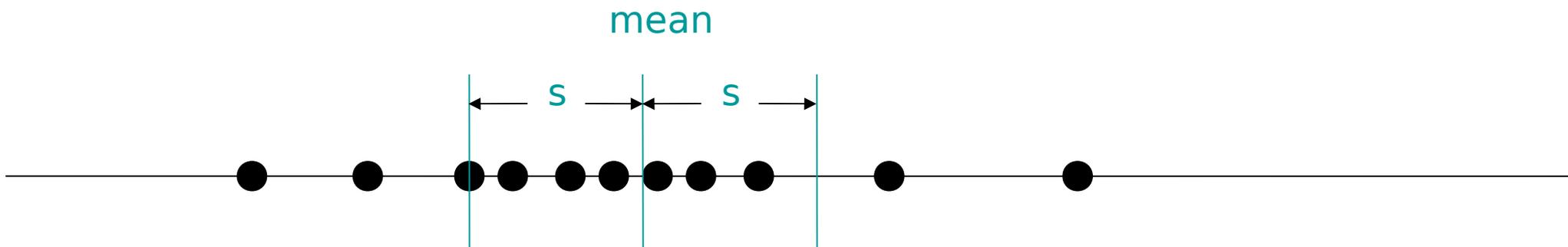
If  $x_1, x_2, \dots, x_n$  are the data in a sample with mean  $m$ , then the sample variance  $s^2$  is:  $s^2 = (\sum(x_i - m)^2) / n$

The larger the variance, the more scattered the observations on average.

*Caveat: many statistical software packages calculate the "bias-corrected sample variance", dividing by  $(n-1)$*

## Standard Deviation

The standard deviation  $s$  is the square root of the variance:  $s = \sqrt{\text{Var}(X)}$



# Quantile, Quartile, Percentile

## Quantile

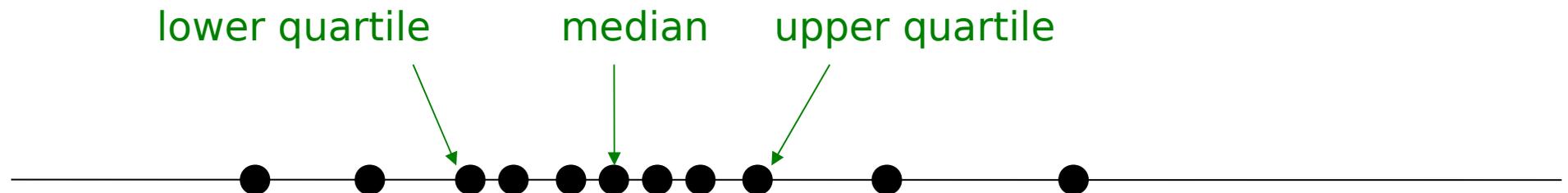
Quantiles are a set of 'cut points' that divide a sample of data into groups containing (as far as possible) equal numbers of observations.

## Quartile

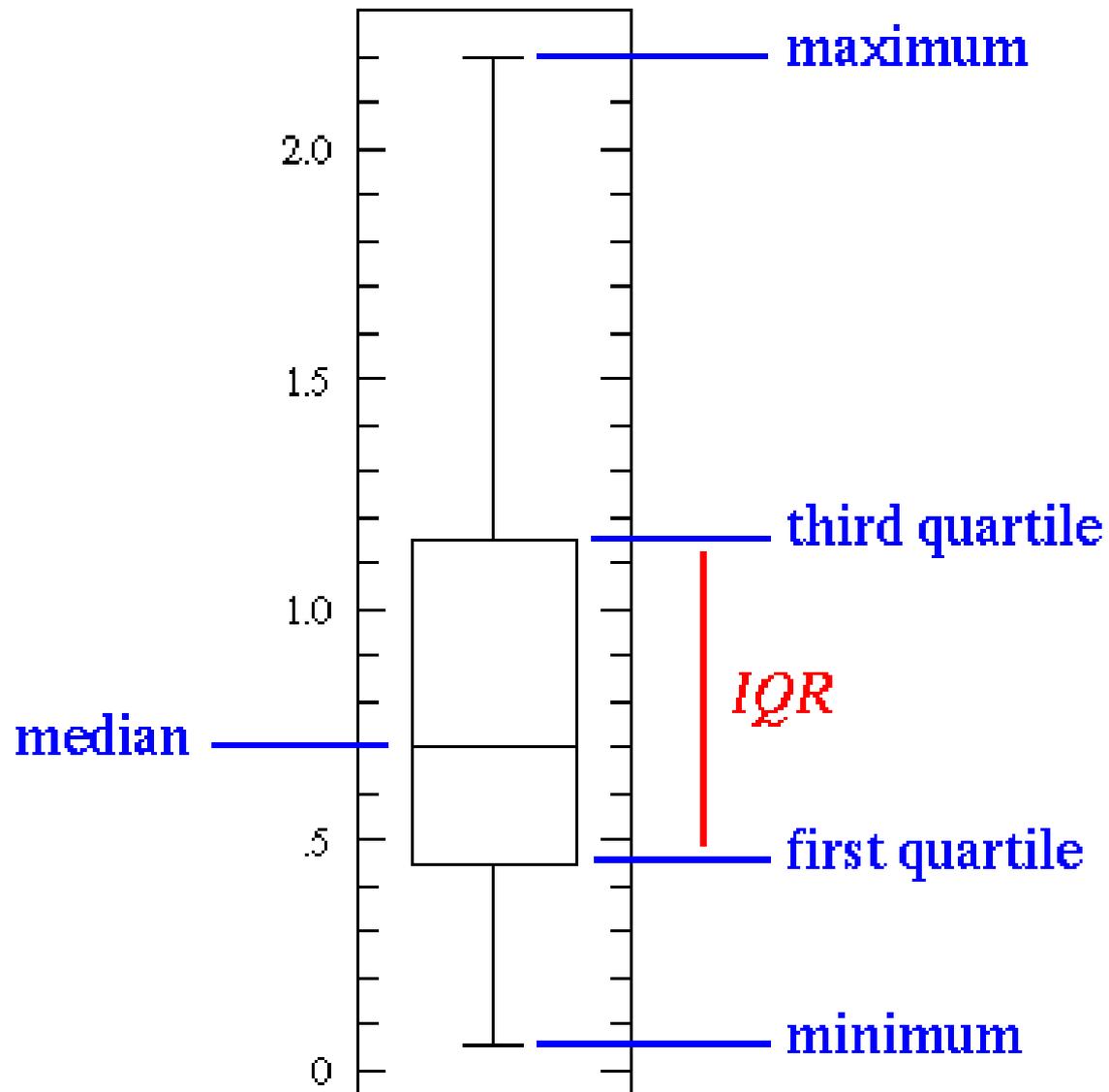
Quartiles are values that divide a sample of data into four groups containing (as far as possible) equal numbers of observations

## Percentile

Quartiles are values that divide a sample of data into hundred groups containing (as far as possible) equal numbers of observations



# Boxplot



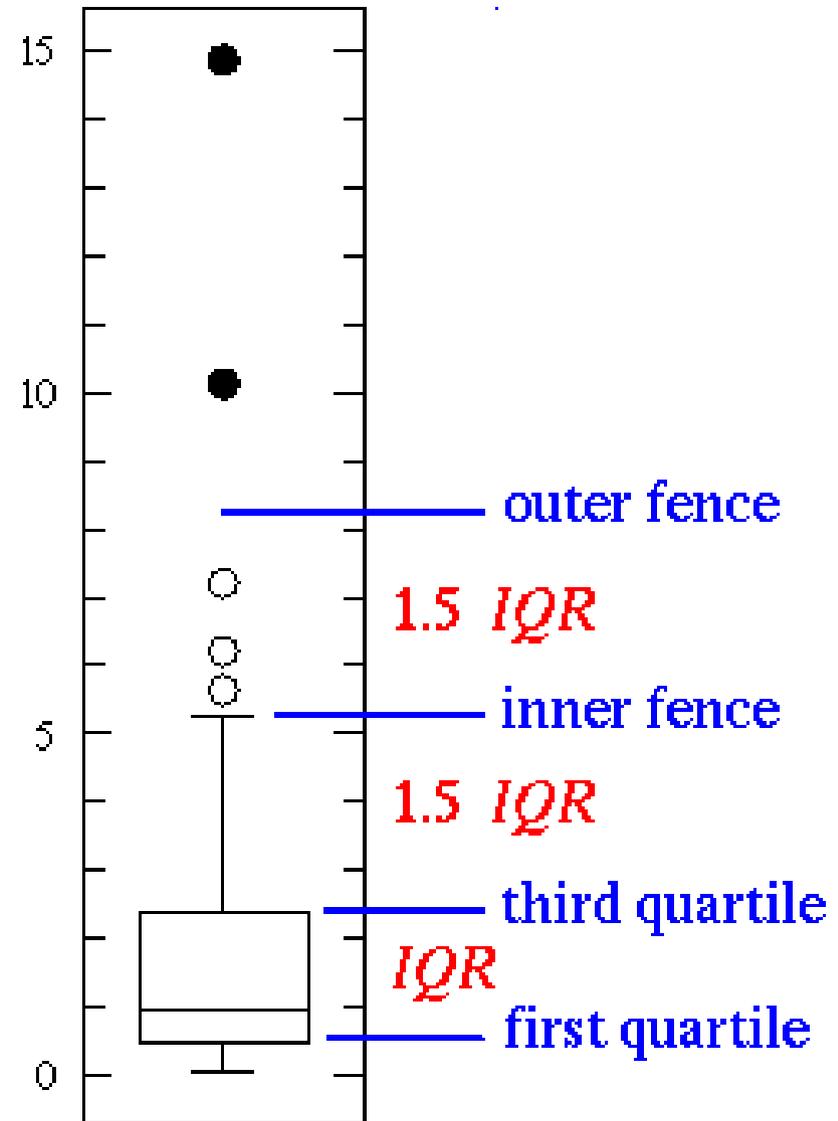
# Outliers

Try to avoid outliers

- Improve your test equipment
  - Eliminate sources of disturbances
  - Repeat parts of your experiment
- in case of disturbance

Outliers are not generally bad – they give you valuable information

With large data sets outliers can often not be avoided



# Some Excel Functions

## MEDIAN(Matrix)

- Matrix            Data row

## QUARTILE(Matrix; Quartil)

- Matrix            Data row
- Quartil            0 = min, 1=lower quartile, 2 = median, 3 = upper quartile, 4 = max.

## QUANTIL(Matrix; Alpha)

- Matrix            Data row
- Alpha            value form 0 to 1.

## Box Plots with Excel 2007

<http://blog.immeria.net/2007/01/box-plot-and-whisker-plots-in-excel.html>

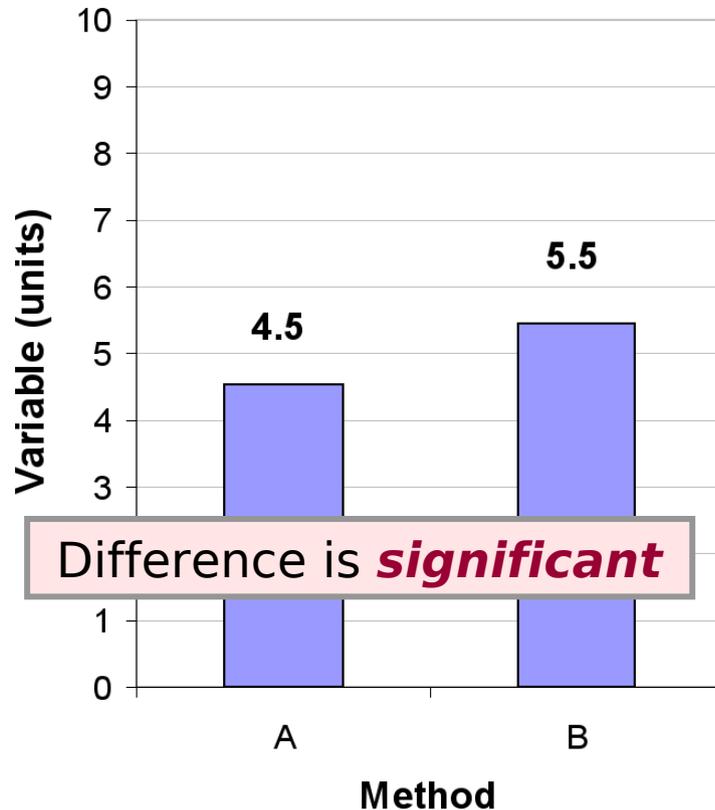
# Don't Do This (II)

“With version A the test users needed 25 seconds in average to complete the task, but with version B it took only 21 seconds. Thus, our user study showed that version B is the better way to solve the task.”

Is the difference significant?

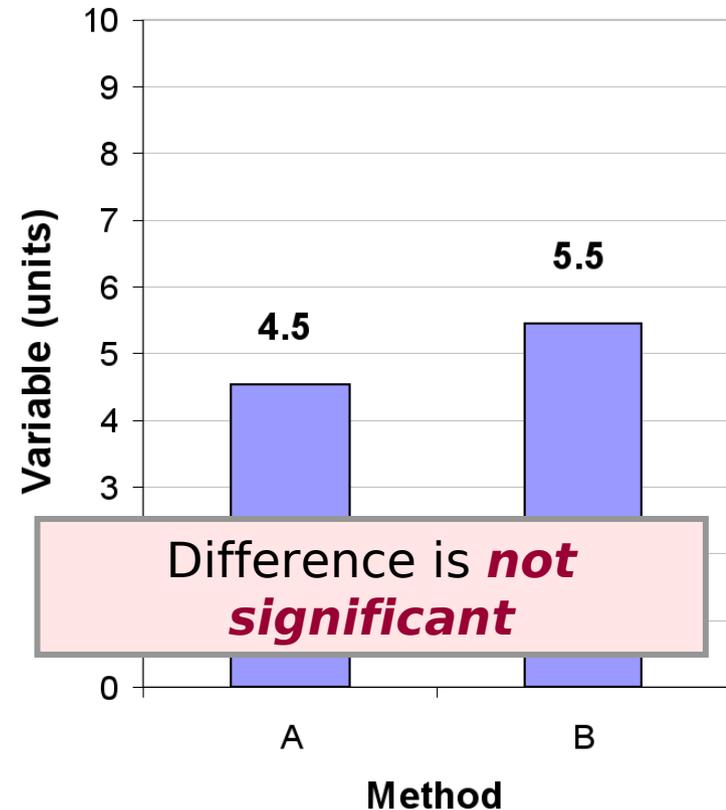
**What does 'significant' mean?**

# Example #1



“Significant” implies that in all likelihood the difference observed is due to the test conditions (Method A vs. Method B).

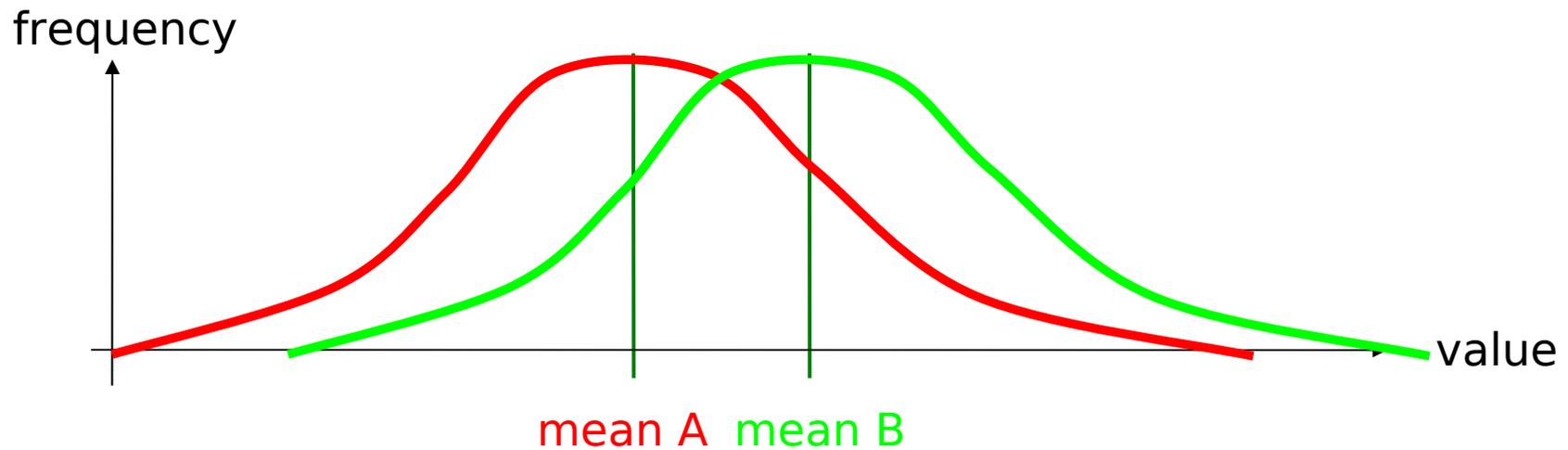
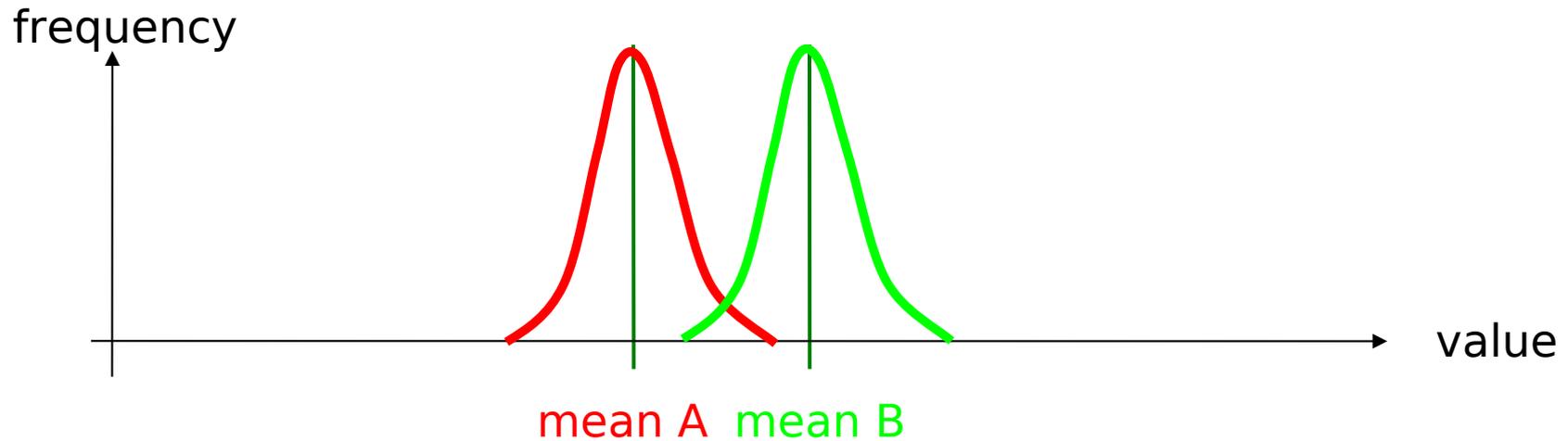
# Example #2



“Not significant” implies that the difference observed is likely due to chance.

# Comparing Values

Significant differences between measurements?



# Significance

In statistics, a result is called significant if it is unlikely to have occurred by chance.

It does not mean that the result is of practical significance!

A statistically significant speed difference of 0.1% between two text-entry methods may have little practical importance.

In the case of hypothesis testing the **significance level** is the probability that the null hypothesis ('no correlation') will be rejected in error although it is true.

Popular levels of significance are 5%, 1% and 0.1%

# Student's t-Test

The t statistic was introduced by William Sealy Gosset for cheaply monitoring the quality of beer brews. "Student" was his pen name. Gosset was a statistician for the Guinness brewery in Dublin.

The t-test is a test of the null hypothesis that the means of two **normally distributed** populations are equal. The t-test gives the probability that both populations have the same mean (and thus their differences are due to random noise).

A result of 0.05 from a t-test is a 5% chance for the same mean.

Different variants of the t-test are used for paired (each sample in population A has a counterpart in population B) and unpaired samples.

Examples:

**Paired:** speed of persons before and after treatment,

**Unpaired:** the reading speed of two different groups of people are compared

(mostly from wikipedia.org)

Student [William Sealy Gosset] (March 1908). "The probable error of a mean". *Biometrika* 6 (1): 1–25.

# Excel: t-Test

Real data from a user study

|             | A             | B             |
|-------------|---------------|---------------|
| K1          | 751           | 1097          |
| K2          | 1007          | 971,5         |
| K3          | 716           | 1121          |
| K4          | 1066,5        | 1096,5        |
| K5          | 871           | 932           |
| K6          | 1256,5        | 926,5         |
| K7          | 957           | 1111          |
| K8          | 1327          | 1211,5        |
| K9          | 1482          | 1062          |
| K10         | 881           | 976           |
| <b>Mean</b> | <b>1031,5</b> | <b>1050,5</b> |

**T-test 0,8236863**

|             | A            | B           |
|-------------|--------------|-------------|
| K1          | 826,5        | 1382        |
| K2          | 806          | 1066        |
| K3          | 791          | 1276,5      |
| K4          | 896,5        | 1352        |
| K5          | 696          | 1191        |
| K6          | 1121         | 1066        |
| K7          | 891          | 1217        |
| K8          | 1327         | 1412        |
| K9          | 1277         | 1266,5      |
| K10         | 656          | 1101        |
| <b>Mean</b> | <b>928,8</b> | <b>1233</b> |

**T-test 0,0020363**

Excel functions used:

```
=MITTELWERT(C4:C13)  
=TTEST(C4:C13;D4:D13;2;1)
```

(function names are localized)  
Menu: Tools>Data Analysis

TTEST(...) Parameters:

- Data row 1
- Data row 2
- Ends (1 or 2) (usually 2)
- Type (1=paired, 2=same variance, 3=different variance)

# T-Test Caveats

- can only be applied to two populations
- do not add significance levels from different tests

# Analysis of Variance (ANOVA)

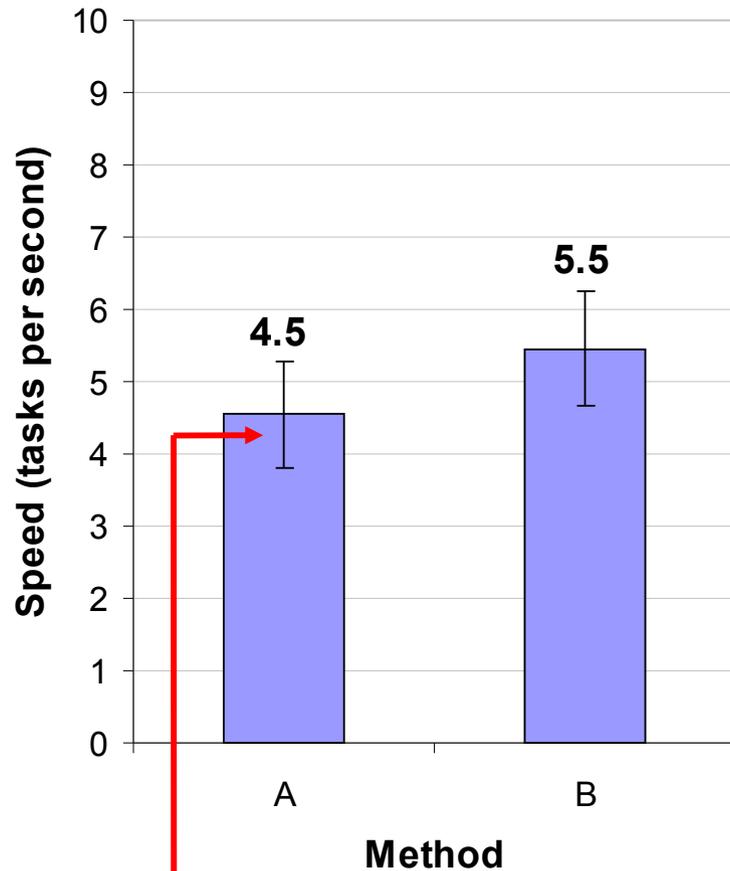
Determine if there is a significant difference between different series of measurements.

“Can the difference be explained by statistical noise?”

## General Concept:

- Calculate the variance within each measurement.
- Calculate the variance in relation to the mean of all series.
- If the variance within a measurement series is much smaller than the variance in relation to the overall mean → significant!

# Example #1 - Details



Error bars show  
 $\pm 1$  standard  
deviation

| Example #1  |        |      |
|-------------|--------|------|
| Participant | Method |      |
|             | A      | B    |
| 1           | 5,3    | 5,7  |
| 2           | 3,6    | 4,6  |
| 3           | 5,2    | 5,1  |
| 4           | 3,3    | 4,5  |
| 5           | 4,6    | 6,0  |
| 6           | 4,1    | 7,0  |
| 7           | 4,0    | 6,0  |
| 8           | 5,0    | 4,6  |
| 9           | 5,2    | 5,5  |
| 10          | 5,1    | 5,6  |
| <i>Mean</i> | 4,5    | 5,5  |
| <i>SD</i>   | 0,73   | 0,78 |

# Example #1 - Anova

ANOVA Table for Speed

|                  | DF | Sum of Squares | Mean Square | F-Value | P-Value | Lambda | Power |
|------------------|----|----------------|-------------|---------|---------|--------|-------|
| Subject          | 9  | 5.839          | .649        |         |         |        |       |
| Method           | 1  | 4.161          | 4.161       | 8.443   | .0174   | 8.443  | .741  |
| Method * Subject | 9  | 4.435          | .493        |         |         |        |       |

Probability that the difference in the means is due to chance

Reported as...

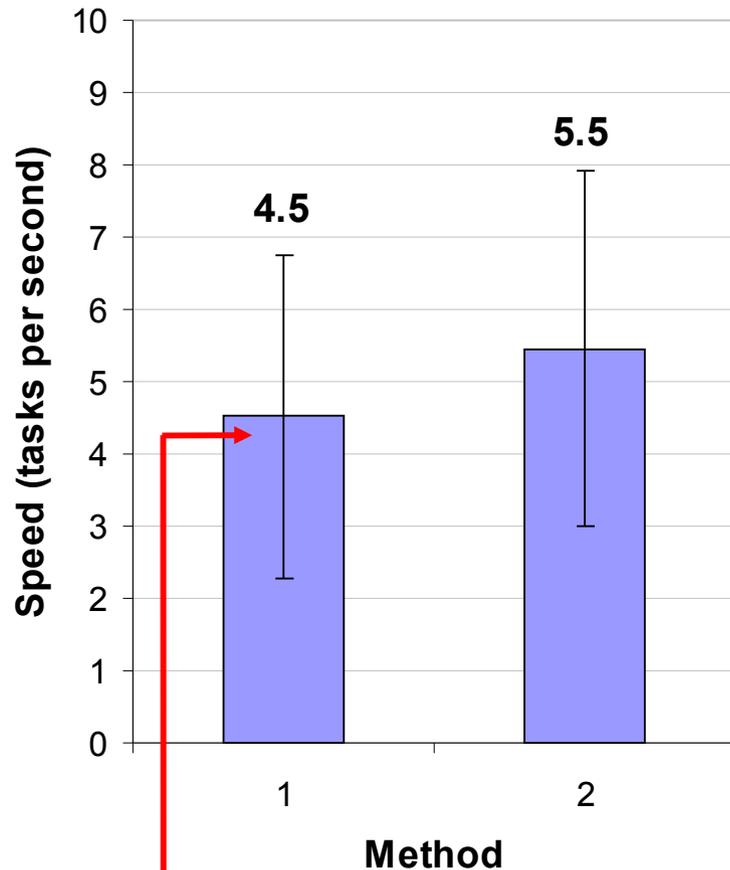
$$F_{1,9} = 8.443, p < .05$$

Thresholds for "p"

- .05
- .01
- .005
- .001
- .0005
- .0001

$$F = \frac{\text{explained variance}}{\text{unexplained variance}}$$

# Example #2 - Details



Error bars show  $\pm 1$  standard deviation

| Example #1  |        |      |
|-------------|--------|------|
| Participant | Method |      |
|             | A      | B    |
| 1           | 5,3    | 5,7  |
| 2           | 3,6    | 4,6  |
| 3           | 5,2    | 5,1  |
| 4           | 3,3    | 4,5  |
| 5           | 4,6    | 6,0  |
| 6           | 4,1    | 7,0  |
| 7           | 4,0    | 6,0  |
| 8           | 5,0    | 4,6  |
| 9           | 5,2    | 5,5  |
| 10          | 5,1    | 5,6  |
| <i>Mean</i> | 4,5    | 5,5  |
| <i>SD</i>   | 0,73   | 0,78 |

# Example #2 – Anova

ANOVA Table for Speed

|                  | DF | Sum of Squares | Mean Square | F-Value | P-Value | Lambda | Power |
|------------------|----|----------------|-------------|---------|---------|--------|-------|
| Subject          | 9  | 37.017         | 4.113       |         |         |        |       |
| Method           | 1  | 4.376          | 4.376       | .634    | .4462   | .634   | .107  |
| Method * Subject | 9  | 62.079         | 6.898       |         |         |        |       |

Probability that the difference in the means is due to chance

Reported as...

$$F_{1,9} = 0.634, ns$$

Note: For non-significant effects, use "ns" if  $F < 1.0$ , or " $p > .05$ " if  $F > 1.0$ .

# Excel: ANOVA

Anova: Single Factor  
**Which Bowler is Best?**  
 SUMMARY

Tools Menu  
 → Data Analysis  
 → One-Way  
 ANOVA

| Groups | Count | Sum  | Average  | Variance |
|--------|-------|------|----------|----------|
| Pat    | 6     | 922  | 153.6667 | 92.26667 |
| Mark   | 6     | 1070 | 178.3333 | 116.6667 |
| Sheri  | 6     | 937  | 156.1667 | 54.96667 |

ANOVA

| Source of Variation | SS       | df | MS       | F               | P-value  | F crit          |
|---------------------|----------|----|----------|-----------------|----------|-----------------|
| Between Groups      | 2212.111 | 2  | 1106.056 | <b>12.57358</b> | 0.000621 | <b>3.682317</b> |
| Within Groups       | 1319.5   | 15 | 87.96667 |                 |          |                 |
| Total               | 3531.611 | 17 |          |                 |          |                 |

Source: <http://www.isixsigma.com/library/content/c021111a.asp>

ANOVA test online: <http://www.physics.csbsju.edu/stats/anova.html>

Java API for ANOVA: <http://www.yorku.ca/mack/RN-Anova.html>

# This Lecture is not Enough!

We strongly recommend to teach yourself.  
There is plenty of material on the WWW.

## Further Literature

Andy Field & Graham Hole: How to design and report experiments, Sage

- Jürgen Bortz: Statistik für Sozialwissenschaftler, Springer
- Christel Weiß: Basiswissen Medizinische Statistik, Springer
- Lothar Sachs, Jürgen Hedderich: Angewandte Statistik, Springer
- various books by Edward R. Tufte